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Examining Technical and Economic Efficiency: Empirical Applications Using Panel Data From Alberta Dairy Farmers.

by

Innocent John Karamagi ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

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Dedication

To
you,
Mama;
your love,
support, and prayers
were a strong ladder I used
to push myself up to the 'roof-top'.

Abstract

This study uses stochastic frontier analysis (SFA) to examine technical and economic efficiency of the Alberta dairy sector. As the North American dairy market moves toward free trade and a more competitive economic environment, the viability of the sector can be maintained if producers are highly efficient, which renders efficiency analysis crucial to production, marketing and trade. Therefore, there is need to emphasise efficiency and management practices that contribute to greater efficiency.

The overall objective was to determine efficiency for milk production in Alberta. Thus, dynamic stochastic production and cost frontiers were estimated and used to compute measures of technical and economic efficiency and to identify factors that have significantly influenced efficiency levels in milk production. Moreover, alternative methodological assumptions about stochastic frontiers were tested, including choice of functional form and alternative distribution assumptions for inefficiency terms.

The data from Alberta Agriculture covering 1980-1996 and constituting 1046 observations was used to generate the variables used.

The findings are; mean technical efficiency, mean economic efficiency and mean allocative efficiency are 91 percent, 84 percent and 93 percent, respectively, with more than 50 percent of farms in the samples performing better than the average farm. Small herds tended to show higher levels of efficiency than large ones; capital intensity, higher breeding and veterinary services, and increase in the ratio of grains and concentrates to hay and forage were found to be associated with higher levels of efficiency.

The output elasticities were all positive and characterised by decreasing returns to scale, implying that Alberta dairy farmers are operating profitably; changes in the levels of technical and economic efficiency over time have not been significant, but technological change has been positive and significant. The stochastic model was found

to be a correct representation of both production and cost frontiers. Lastly, different distributions of the inefficiency error term were found to be inconsistent in terms of resulting average measures and the ranking of individual technical and economic efficiency measures, indicating that the appropriate choice of the distribution is dependent on the data. Future research should utilize data from several provinces under one study to afford meaningful comparisons.

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Chapter 1. Introduction

1.1 Background and Economic Problem

This study examines technical and economic efficiency of Alberta dairy farms. The impetus for this examination arises from two sets of related issues. The first set of issues, discussed in the background below, concerns the role that dairy farming plays in the provincial economy of Alberta, and the changes in the technological, regulatory and international trade environments facing Alberta dairy producers. The second set of issues, identified as the economic problem of this study, centres on how the issues discussed in the background are likely to affect the Alberta dairy sector in terms of an increased emphasis on productive efficiency in the sector.

1.1.1 Background

Dairy farming in Alberta ranks high in agricultural contribution to the province's economy. In 1999, it ranked as the fourth most important activity in terms of total farm gate income, generating $327 million, or 6.2 percent of the farm cash receipts of the province. The province's 953 dairy farmers produced in the 1999-2000 dairy year 6.2 million hectolitres of milk, of which 44 percent was used for industrial requirements and 56 percent to meet fluid demand (Canadian Dairy Commission, 2000). This output ranks Alberta as third behind Quebec and Ontario in milk production, accounting for 8 percent of the estimated Canadian dairy herd. In addition, Alberta's dairy processing sector contributes approximately 10 percent of the total value of Alberta food and beverage production, with total dairy exports valued at $18 million (Growing Alberta, 2000)[1].

---

[1] For an assessment of the economic importance of the Canadian dairy sector, see Agriculture and Agri-Food Canada (1996).

In order to keep their sector viable Alberta dairy farmers must be able to compete in the changing technological, structural and economic environment. They need to keep pace with and adjust to the changes that have been taking place in the dairy sector. These include technological innovations, adjustments in the marketing system and in international trade agreements. Technological changes have encompassed advancement in record keeping, breeding, feeding and milking systems, and nutrition. These changes have contributed to a sharp decrease in the number of farms, an increase in the average herd size, increased farms' specialisation in milk production, a rise in the proportion of high-yielding breeds, more mechanisation and high quality breeding (Fox et al. 1992).

The impact on the size of the dairy sector has been significant. In Canada, over the last 25 years approximately one farm out of six has remained in the sector, with small dairy farms showing the greatest decline in numbers (Agriculture and Agri-Food Canada, 1996). In Alberta, the number of dairy cows has decreased from 250,000 in 1965 to less than 100,000 in 1998, while total milk production has doubled over the same period from 300 million litres to almost 600 million litres. This translates to a five-fold increase in milk per cow (Cameron & Gould, 1998).

In terms of the international trade environment, there are significant prospects for gradual elimination of import controls for dairy products. These changes are provided for under both the General Agreements on Tariffs and Trade (GATT)/World Trade Organisation (WTO) and the North American Free Trade Agreements (NAFTA) (Barichello et al. 1996). The GATT/WTO Uruguay Round Agreements (URA), for example, required a shift away from non-tariff barriers, which for Canada implied a shift from import quotas to tariffs. The Canadian dairy sector is affected by these regulations because it is one of the most protected sectors in Canadian agriculture. In 1996, the representatives of Canadian dairy farmers successfully defended the existing tariff rates on imports of milk and milk products against a U.S. challenge before GATT/WTO. However, with the implementation of NAFTA, farm programmes are being increasingly criticised (Amara et al., 1999) With further pressure for tariff reduction under NAFTA, tariffs may decrease more quickly, thereby exposing Canadian industry to competition from the U.S. (Barichello et al., 1996). Furthermore, the U.S. launched a GATT/WTO challenge of Canada's new dairy export policy in August 2000[2]. While the WTO initially ruled in favour of the U.S. in July 2001, in December 2001 Canada won an appeal against the WTO ruling.[3] As Cordon (2002) notes, the case called into question whether any government can maintain a regulated domestic market for agricultural products without violating international trade rules.

In addition to the changes in the international environment, there have also been changes in the domestic policy environment. In 1995, Canadian dairy stakeholders implemented regional pooling of market returns, administered by the Canadian Dairy Commission[4]. Further deregulation may entail the inter-provincial transfer of quotas, enhancement of multiple-component pricing, and ultimately, the complete removal of barriers to inter-provincial trade. Wholesale pricing is already deregulated to the extent that milk pricing is no longer regulated beyond the farm gate. Furthermore, because of budgetary restraint, the federal subsidy to producers for industrial milk was phased out, being finally eliminated on January 31, 2002 (Canadian Dairy Commission, 2002).

Not withstanding the significant productivity growth that has occurred in the Canadian dairy industry, this productivity growth would have been even greater in the absence of supply management policies. For example, evidence shows that supply management has slowed the growth of the Alberta dairy sector (Richards, 1993). In this regard, Richards observes that any intervention that slows the rate of productivity growth will cause the production costs to be higher than they would otherwise be, thereby ensuring inefficiency in production.

---

[2] This new export policy was geared at encouraging exporters to contract directly with producers outside the control of provincial marketing boards.

[3] If the original ruling had been upheld, the US and New Zealand would have been allowed to levy more than $ 1 billion trade sanctions against Canada (Cordon, 2002).

[4] The regions are divided into two pools, namely the Eastern and Western pools. The Eastern pool include the provinces of Manitoba, Ontario, Quebec, New Brunswick, Nova Scotia, and Prince Edward Island and the Western pool include the provinces of Saskatchewan, Alberta, and British Columbia, and Northwest Territories.

Regulatory and technology influences have contributed to structural changes in the Alberta dairy sector over time; specifically, they have led to fewer farms with larger herds and more productive cows. In Alberta, although the provincial average daily milk shipment has steadily increased, the number of fluid milk producers has continued to decline. In 1995-96, there was a 5% decrease in the number of producers, with a further 4.25% decline occurring in 1996-97. However, the remaining producers increased their milk shipments by 7.89% and 7.08% in 1995-96 and 1996-97, respectively. The Alberta Dairy Control Board registered and licensed 37 new producers in 1995-96, in spite of the net decline in the total number. Moreover, overall production increased from 571 million litres to 586 million litres in 1995-96, which increased further to 608 million litres in 1997-98. The number of producers declined further by 8%, as shipment per producer increased from 1,557 to 1,706 litres per day in 1997-98 (Alberta Dairy Control Board, 1996, 1997,1998).

1.1.2 The Economic Problem

The regulatory and technological changes faced by dairy producers are likely to continue into the future. North America and the North American market in dairy products are gradually moving toward free trade. With reduced protection, the ability to sustain the Canadian dairy sector will depend on the producers' competitiveness (Richards & Jeffrey, 1996). Barichello et al., (1996) contend that the Canadian dairy sector will undergo a significant rationalisation if cost improvements are not made over the period allowed by GATT/WTO for tariff reduction. This will result in an increased share of output being produced by regions that are able to successfully compete with producers in other countries and other provinces. In a more competitive environment, the viability of the sector can be maintained if producers are highly efficient.

Both technical and economic efficiency are crucial to production, marketing and trade. Free trade and other changes will work in favour of the more efficient dairy producers and processors. This will result in an increased need to emphasise efficiency and management practices that contribute to greater efficiency.

Alberta dairy farmers need to address a two-fold question: how efficient are their farming practices and what factors determine their levels of efficiency? To date, information addressing efficiency and management practices of Alberta dairy farmers has been insufficient. First, most applied studies on efficiency in dairy production have not dealt with Alberta. Second, most of those that have dealt with Alberta are not rigorous in their approach (e.g., Jeffrey, 1992; Barichello et al., 1996). Those that have been rigorous have focused only on technical efficiency (e.g., Jeffrey and Richards, 1996). Lastly, studies have tended to rely on a limited sample or time frame (e.g., Jeffrey & Richards, 2000). As a result, there is need to examine both technical and economic efficiency in Alberta production, and to identify management factors that have influenced efficiency in the extended time frame.

1.2 Objectives and Significance of the Study

The overall objective of this study is to determine the efficiency – technical as well as economic – for milk production in Alberta.

The specific objectives of this study include the following:

- to estimate dynamic production and cost frontiers for Alberta dairy production;

- to compute technical and economic efficiency and establish their degree of variability among a sample;
- to identify factors that have influenced technical and economic efficiency in milk production of Alberta dairy farmers;
- to test for the optimality of scale of operation.
- to test the impact of different methodological assumptions about stochastic frontiers; specifically,

a) The choice of functional forms that represent production and cost frontiers for Alberta dairy farms;

b) A comparison of technical and economic efficiency levels from different estimations;

c) A test of the extent to which random factors account for production and cost of Alberta dairy milk production, that is, the extent to which the frontiers depart from the respective deterministic kernels;

d) Examination of the effect of alternative distribution assumptions of the inefficiency error term on the estimated frontiers, efficiency levels and performance ranking of farms.

The results of this study may be useful in several respects. Alberta dairy producers may use recommendations concerning technical and socio-economic factors that are important for efficient dairy production to assess their own management practices relative to "efficient" practices. This may lead to improved levels of efficiency. Moreover, to maintain a viable dairy sector, the Alberta producers will need an edge over economic rivals in both technical and economic efficiency. By complementing information contained in other studies, this study may help to indicate how the Alberta dairy producers fare in comparison with potential competitors.

In addition, none of the previous studies that have analysed productive efficiency for Alberta has tested the methodological assumptions noted above. Therefore, the results of the study will add to and complement those of studies that have approached the producers' efficiency in a static setting in analysing Alberta dairy producers and by shedding more empirical light on some methodological assumptions related to efficiency analysis.

1.3 Organisation of the Study

The study is organised in five chapters as follows. Chapter 2 discusses technical and economic efficiency from the theoretical and analytical perspectives. Initial definitions of efficiency are provided, followed by reviews of the theoretical framework for both technical and economic efficiency. The chapter finishes with a review of empirical studies in Canada and the U.S. Chapter 3 provides a detailed discussion of the empirical methods used in the study, elaborating on the models and pertinent methodological issues. The first part develops the stochastic models for production and cost frontiers, as well as the measures of technical and economic efficiency, using econometric techniques. The second part of the chapter provides a discussion related to the data used in the study. In Chapter 4, the results from the estimation of models are presented and discussed. Chapter 5 provides a summary, recommendations and conclusions of the study.

Chapter 2. Technical and Economic Efficiency: Theoretical and Analytical Framework

2.1 Definitions of Efficiency

The examination of efficiency dates back to 1951 in the works of Debreu and Koopmans. While Koopmans (1951) provided a definition of technical efficiency, Debreu (1951) introduced its first measure with the "coefficient of resource utilization". Following on Debreu, Farrell (1957) developed a rigorous method of measuring relative technical and economic efficiency. For this, he estimated the production frontier, a function for "fully efficient" firms in that they produce maximum output, given a certain amount of inputs[5].

Farrell argued that the efficiency of a firm constitutes two components: technical efficiency and allocative efficiency. Technical efficiency (TE) reflects the ability of a firm to obtain maximum output from a given set of inputs. Hence, technical inefficiency refers to the inability of a firm to use a set of inputs to generate the highest attainable output from those inputs. In other words, the firm fails to produce at the outer bound of its production function. Allocative efficiency (AE) reflects the ability of a firm to use inputs in optimal proportions, given their respective prices. Allocative inefficiency therefore arises when a firm fails to take advantage of using substitutable cheaper inputs to incur the minimum cost of production. A firm's efficiency may be a combined effect of TE and AE. This combined effect is termed economic efficiency (EE), and is measured as a product of TE and AE.

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[5] Alternatively we may say that the efficient firm uses minimum levels of inputs to produce a stipulated level of output.

2.2 Theoretical Basis for Technical Efficiency and Economic Efficiency

2.2.1 Technical Efficiency

The basis for technical efficiency is producer theory. Consider a producer employing a number of inputs $\mathbf{x} \equiv (x_1, \ldots x_n)$ that it purchases at given input prices $\mathbf{w} \equiv (w_1 \ldots w_n) > 0$ to produce a single output *q* that it sells at a fixed price p>0. This producer would transform inputs into output efficiently along a production function f(**x**), a function that shows maximum output obtainable from the inputs used in production. This function f(**x**) is the production frontier, as it characterizes output-maximizing behaviour of an efficient producer, thereby placing theoretical limits on the possible values of the function (Fϕrsund et al., 1980).

If a firm produced its output with a production plan (q^0, x^0), such a plan would be termed technically efficient if $q^0 = f(x^0)$ and technically inefficient if $q^0 < f(x^0)$. Thus, given the production of efficient firms, $f(x^0)$, TE would be measured theoretically as $0 < \frac{q^0}{f(x^0)} \leq 1$. In practice, however, we estimate the production of efficient firms (i.e., firms operating on the production frontier) from sample data.[6] Thus, technical efficiency is the ratio of a firm's mean output to the corresponding mean potential output (i.e., its mean output if it were to utilise the levels of inputs efficiently), conditional on both the levels of factor inputs being used and inefficiency effects (Battese & Coelli, 1988). Simply stated, it is the ratio of the observed output for the firm, relative to the potential output defined by $f(x^0)$.

---

[6] According to Coelli (1995b), estimating the production function directly is justified if we assume that 1) the input levels are fixed and the firm's management is attempting to maximize output given these inputs; or 2) the management is selecting the levels of inputs and output to maximize expected (rather than actual) profit.

2.2.2 Economic Efficiency

Measurement of economic efficiency is coupled with TE and AE, as introduced by Farrell (1957). Farrell's measures of economic efficiency are based on the unit isoquant, which defines the input ratios associated with the efficient input usage in producing a unit of output, thereby representing a production frontier. Farrell (1957) argued that if the production frontier had constant returns to scale, then the observed input-per-unit of output values for technically inefficient firms would be above the unit isoquant.

Moreover, since along any given isoquant the firm will be *allocatively* efficient if it combines inputs to produce output in a way that minimizes its cost, Farrell associated deviations from the cost-minimizing input ratios (along any particular isoquant) with allocative inefficiency. A given combination of inputs and output is therefore *economically* efficient if it is both technically and allocatively efficient; that is, when the related input ratio is on both the isoquant and the expansion path.

A technically efficient firm may not necessarily minimize its cost of production; hence it may not necessarily achieve economic efficiency. If the firm uses inputs without paying regard to their relative prices, it may achieve technical efficiency, but it may not achieve allocative efficiency (Richards & Jeffrey, 1996). This contention is clarified with the aid of a diagram below.

In Figure 2.1, the curve SS' is an isoquant, representing technically efficient combinations of inputs, x_1 and x_2, used in producing output q. WW' is an isocost line, which shows all combinations of inputs x_1 and x_2 such that input costs sum to the same total cost of production. Since the efficient isoquant represents the production frontier, all points on SS' are technically efficient. As well as being technically efficient, point E on the isoquant is also allocatively efficient, as it represents the least cost feasible combination of x_1 and x_2 in the production of q. Hence at point E the producer is *economically* efficient.

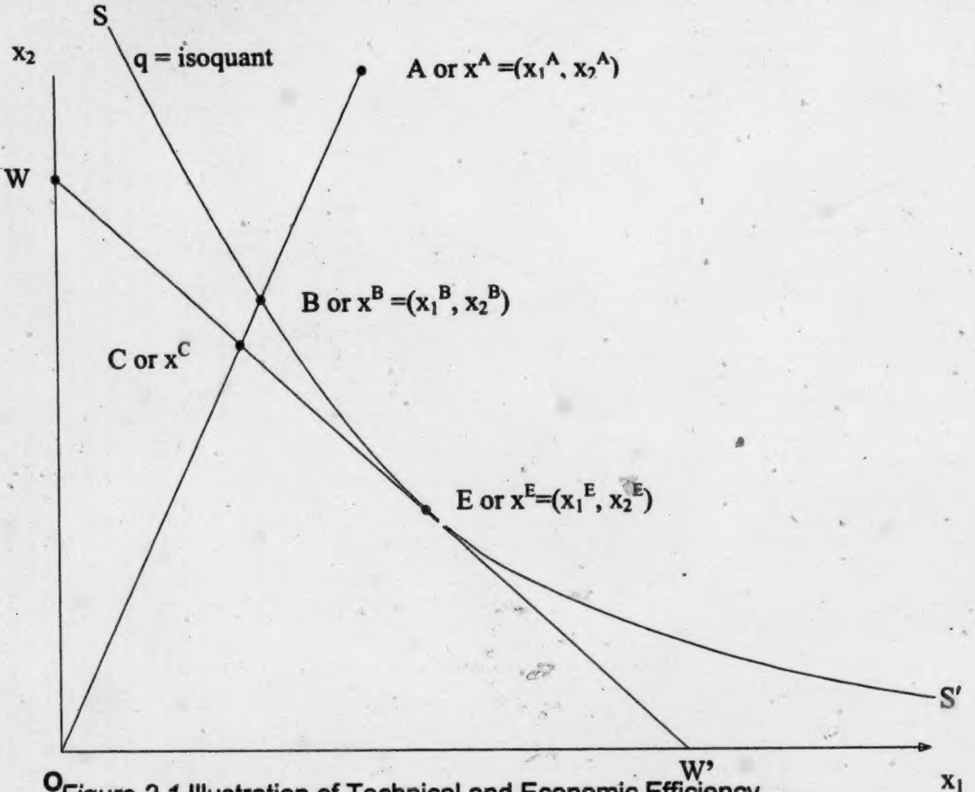
S
x_2
q = isoquant
A or $x^A = (x_1^A, x_2^A)$
W
B or $x^B = (x_1^B, x_2^B)$
C or x^C
E or $x^E = (x_1^E, x_2^E)$
S'
O
W'
x_1

Figure 2.1 Illustration of Technical and Economic Efficiency

How do we measure technical, allocative, and economic efficiency? Suppose a firm is producing its output as depicted by the isoquant SS' with input combination depicted by point A in Figure 2.1. At this point production is neither technically nor allocatively efficient. The degree of technical efficiency for this producer is given by the ratio OB/OA. The distance between B and A represents the proportional reduction in all inputs used in production that could theoretically be achieved without any reduction in output. The producer's degree of allocative efficiency is measured by the ratio OC/OB, which follows from the interpretation of CB, the distance along the ray OA between the isoquant and the isocost line. This distance is the proportion of cost that the producer would save if fully efficient at E, rather than technically efficient, but allocatively inefficient, at B. Hence, CB/OB represents the extent of cost reduction from reallocating inputs to eliminate allocative inefficiency at point B. In other words, OC/OB is the level of allocative efficiency.

The degree of economic efficiency for the producer at A is given by the ratio OC/OA. This measure follows from interpretation of the distance CA as the reduction in cost that would occur if a technically and allocatively inefficient producer at A were to become efficient (both technically and allocatively) at E. It is equally measured as the product of technical and allocative efficiency: $EE = \frac{OB}{OA} \cdot \frac{OC}{OB} = \frac{OC}{OA}$.

The measurement of economic efficiency necessitates the use of dual forms of the production technology, such as the indirect cost or profit function. These forms reflect alternative behavioural objectives (i.e., cost minimisation or profit maximisation) and can account for multiple outputs (Greene, 1993; Coelli, 1995a).

The cost function $c(q, w) \equiv \min_x \{w'x \mid f(x) \geq q, x \geq 0\}$ represents efficient production technology, under certain regularity conditions (as a profit function equivalently does under different regularity conditions).[7] This function shows the minimum money outlay that is required to produce output q if input prices are given by the vector **w**. By invoking Shephard's Lemma, factor demand functions $x_i*(q,w)$, *i*=*1*, 2, ...*n* are obtained from $\bar{v}_w c(q,w^A) \equiv \left[\frac{\partial c}{\partial w_1^A}, \frac{\partial c}{\partial w_2^A}, \ldots, \frac{\partial c}{\partial w_n^A}\right]$, a vector of derivatives of the cost function with respect to each input price, i.e., $x*(q,w) \equiv \bar{v}_w c(q,w)$, provided that the derivatives exist. The factor demand functions $x*(q,w)$ represent the level of input use by an economically efficient producer to produce a given amount of output *q* if input prices are **w**.

To be consistent with relevant economic theory, the cost function must satisfy certain regularity conditions. Specifically, the cost function is non-decreasing in output, linearly homogeneous in input prices, monotonically increasing in prices, and concave and continuous in prices. The last condition implies that the Hessian matrix of the demand functions must be negative semi-definite, which in turn implies that the input demand functions are negatively sloped and the cross price elasticities are symmetrical (Chambers, 1988).

A firm may fail to minimize its costs of production by being technically inefficient, allocatively inefficient or both. If the firm uses inputs excessively, operating at the wrong scale, it will spend more than is necessary, without getting maximum output from the level of inputs used. Such a firm is not minimizing its costs (i.e., $w'x^0 > c(q^0, w)$), and it is technically inefficient. If the firm uses inputs in the wrong proportions, relative to what would be suggested by optimising conditions, it will fail to minimize costs of production, and will certainly be allocatively and economically inefficient.

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[7] Since milk production in Alberta is controlled by a system of milk supply-management quotas, it is plausible to assume, at least in the short run, that farmers minimize cost subject to a fixed output constraint (Moschini, 1988; Richards & Jeffrey, 1998). In addition, the fact that cost minimization requires input price exogeneity, the analysis entails competitive firm behavior.

The firm with a production plan (q^0, x^0) is allocatively efficient when $\frac{f_i(x^o)}{f_j(x^o)} = -\frac{w_i}{w_j}$ $\forall$ *i,j=1,2…n i≠j* (assuming f is differentiable); that is, when the marginal rate of technical substitution between each pair of inputs is equal to the negative ratio of the corresponding input prices. Allocative inefficiency therefore implies $\frac{f_i(x^o)}{f_j(x^o)} \neq -\frac{w_i}{w_j}$. Since a firm is both technically and allocatively efficient when the observed cost equals the minimum cost $c(q^0, w)$ then at that point the observed input usage $\mathbf{x^0}$ equals the cost-minimizing input demand $x_i^*(q^0, w)$. A combination of technical and allocative inefficiency causes $x_i^0 \geq x_i^*(q^0, w)$ for some inputs, but it may cause $x_i^0 \leq x_i^*(q^0, w)$ for other inputs.

2.3 Approaches to Measuring Efficiency

Farrell's (1957) work gave rise to a proliferation of studies that have applied, extended or refined frontier modelling.[8] Bauer (1990) attributes the widespread use of frontier modelling to three factors. First, a frontier is consistent with the underlying theory of optimising behaviour. Second, deviations from the frontier are interpreted as a measure of the efficiency with which economic units pursue their technical or behavioural objectives. Third, information about the structure of the frontier and about the relative efficiency of economic units is useful for policy analysis.

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[8] Since Farrell's (1957) original work, hundreds of studies have been undertaken using a variety of approaches. These studies have typically involved the use of either cross sectional or panel data to measure TE and/or AE. Studies that have estimated stochastic frontiers have made a number of distributional assumptions for the random variables involved and have considered various estimators for the parameters of the models. A survey of these studies is provided in papers by Fϕrsund et al. (1980), Battese (1992), Bravo-Ureta and Pinheiro (1993), and Coelli (1995a).

In estimating frontiers and measuring efficiency, researchers have taken either a parametric or non-parametric approach, using either deterministic or stochastic estimation methods. The parametric and non-parametric approaches differ in three respects. First, the non-parametric approach does not impose a functional form on the data. Second, it does not make assumptions about the distribution of the error term that represents inefficiency. Lastly, the estimated non-parametric frontiers have no statistical properties on which to be gauged.

Deterministic estimation methods attribute any deviations from the frontier as resulting solely from inefficiency. Inefficiency of a firm in this respect is therefore defined as the proportion by which the level of production is less than the estimated frontier output. By failing to account for the possibility of random influence, deterministic frontiers are particularly sensitive to outliers and measurement errors. Conversely, stochastic estimation methods involve specification of a probabilistic frontier that takes into account the possibility of variation in output due to factors not under the firm's control (e.g., measurement errors, weather, disruption of supplies, topography, etc.).

Farrell's original approach was deterministic and non-parametric. This simple linear programming method was extended and applied by Farrell and Fieldhouse (1962), Seitz (1970, 1971) and Todd (1971). Charnes et al. (1978) extended and refined this approach into what is now referred to as Data Envelope Analysis (DEA). With DEA, a researcher constructs a non-parametric frontier that envelops the data points such that all observed points, for example, lie on or above the cost frontier, or on or below the production frontier.

Coelli (1995a) observes that DEA methods have been extended and applied in a large number of papers, especially in management science and service industries.[9] These methods, in addition to having the advantages of the non-parametric approach, enable one to estimate efficiency for multiple–input multiple-output technologies. In agricultural economics applications, however, DEA methods have not been as popular as stochastic frontier analysis (SFA)[10].

2.3.1 Production Frontiers (Measuring TE)

Aigner and Chu (1968) introduced a deterministic method for estimating parametric production frontiers, by estimating a Cobb-Douglas production frontier using mathematical programming techniques. This entailed minimizing the sum of absolute residuals (linear programming) or the sum of squared residuals (quadratic programming) subject to the constraint that all residuals be negative. However, the frontier they estimated was supported only by a subset of data and was therefore sensitive to outliers. Aigner and Chu suggested that one could solve this problem by discarding a few observations.

Timmer's (1971) solution to this problem was to drop a percentage of firms closest to the estimated frontier in order to reduce impact of outliers. This approach has had few followers to date, probably due to the arbitrary nature by which the observations to be omitted are selected (Coelli, 1995a).

Afriat (1972) made assumptions about $\mathbf{x}$ (the vector of explanatory variables) and u (the error term of the production function) and used econometric techniques to estimate a deterministic statistical frontier.[11] Afriat assumed *u* to be independently and identically distributed (iid) and **x** to be independent of *u*. In addition, he suggested a two-parameter beta distribution for *u* and the estimation of the model by maximum likelihood (ML) methods.

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[9] Details of DEA and a review of studies that have utilized it are found in Seiford and Thrall (1990) and Ali and Seiford (1993). See also Coelli (1995a) for a short discussion of DEA.

[10] The reviews by Battese (1992), and Bravo-Ureta and Pinheiro (1993) show a lack of adoption of DEA in the agricultural economics literature. A survey of studies between 1985 to 1994 by Coelli (1995a) on applications of frontiers to agriculture constituting a total of 15 countries lists 27 stochastic frontier and only 3 DEA applications.

With the functional form for the deterministic model specified, a set of parameter estimates, β, for the frontier is estimated by ML estimators or by corrected ordinary least squares (COLS) (Richmond, 1974). However, with the exponential or half normal distributions for the u_i's, one cannot make inferences about the β parameters from maximum likelihood estimates (Schmidt, 1976; Greene, 1980). This is because the process violates the regularity conditions, which ensure that estimators obtained using maximum likelihood methods are consistent and asymptotically efficient.

As noted earlier, deterministic methods attribute all deviations to inefficiency, which is a potentially limiting assumption. This limitation is addressed by stochastic parametric frontier methods (Aigner et al., 1977; Meeusen & van den Broeck, 1977). The error term of the stochastic model is assumed to have two additive components: a symmetric component accounting for pure random factors and a one-sided component that captures the effects of inefficiency relative to the stochastic frontier. Moreover, the model meets regularity conditions, which permits estimation of standard errors and tests of hypotheses. As a result of these properties and advantages, this study uses stochastic estimation methods to examine productive efficiency for Alberta dairy farms.

For a production frontier, the Aigner et al. (1977) stochastic model is defined, for a sample of N firms, as

$$Q_i = f(x_i;\beta)e^{(\varepsilon_i)} \qquad (2.1)$$

---

[11] Aigner and Chu's (1968) model, although parametric, did not make explicit these assumptions. Hence, no statistical properties for the "estimators" resulted from it (Schmidt, 1976).

where Q_i is the possible (i.e., efficient) production level, which is bounded by the stochastic quantity $f(x_i;\beta)e^{v_i}$; $f(x_i;\beta)$ is a suitable function for a vector x_i of factor inputs and other explanatory variables associated with the ith firm and a vector β of unknown parameters, and $\varepsilon_i = v_i - u_i$, where the v_i's are errors accounting for random variation in output across firms due to factors beyond the firm's control. These errors are assumed to be $N(0,\sigma_v^2)$ random variables[12] and independent of the u_i's. The u_i's are errors that reflect technical inefficiency relative to the stochastic frontier; they are typically assumed to follow either an exponential distribution or a half-normal distribution (i.e., non-negative truncation of the $N(0,\sigma_u^2)$ distribution), where technical inefficiency of the ith firm is defined as

$$TE_i = \frac{y_i}{\exp(x_i\beta)} = \frac{\exp(x_i\beta - u_i)}{\exp(x_i\beta)} = \exp(u_i) \tag{2.2}$$

The estimated efficiency measures from stochastic frontier analysis are sensitive to the assumed distribution for u_i. Meeusen and van den Broeck (1977) assumed u_i's to have an exponential distribution and showed that their model was not as restrictive as the one-parameter gamma distribution considered by Richmond (1974). Still, researchers have not been able to justify *a priori* the selection of any particular distributional form for u_i. To alleviate this problem, some have specified distributions that are more general, the most common ones being the truncated normal (Stevenson, 1980) and the two-parameter gamma (Greene, 1990). These distributions and the sensitivity of efficiency measures to alternative distributional assumption are discussed further in the next section (Sec. 2.3.2).

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[12] Stochastic variation in output is reflected in the magnitude of the variance, σ_v^2. If σ_v^2 were 0, the frontier would be deterministic.

In a majority of cases, empirical studies have used the half-normal specification for u_i, consistent with the Aigner et al. (1977) model (Coelli, 1995a; Bauer, 1990; Bravo-Ureta & Pinheiro, 1993). Aigner et al. (1977) derived the log-likelihood function for their model in equation (2.1) under the assumptions outlined above. Because the standard regularity conditions hold (i.e., the conditions that are required to ensure that estimators obtained using maximum likelihood methods are consistent and asymptotically efficient), Aigner et al. (1977) used the maximum likelihood (ML) estimators to make inference about the parameters of the model. In computing the estimates, Aigner et al. (1977) expressed the likelihood function in terms of the variance parameters,

$$\sigma^2 = \sigma_u^2 + \sigma_v^2 \qquad (2.3)$$

and $$\lambda = \frac{\sigma_u^2}{\sigma_v^2} \qquad (2.4)$$

with the truncated half-normal distribution for u_i and the assumed symmetric distribution for v_i. This likelihood function is given as

$$\ln L = \frac{N}{2}\ln(\pi/2) - N\ln\sigma + \sum_{i+1}^{N}\ln\left[1-\phi(-\frac{\varepsilon_i\lambda}{\sigma})\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\varepsilon_i^2 \qquad (2.5)$$

where N is the number of observations, $\varepsilon_i = v_i - u_i$, consistent with the overall error term in (2.1), $\phi(.)$ is the standard normal probability density function, and σ, λ are defined as before. However, λ is very large σ_u^2 is large relative to σ_v^2, and in the extreme when $\sigma_v^2 = 0$ λ will be equal to infinity. Since λ is non-negative, Battese and Cora (1977) replaced it with

$$\gamma = \frac{\sigma_u^2}{\sigma^2} \qquad (2.6)$$

the values of which lie between zero and one. This provides the opportunity to search the parameter space of γ for a suitable starting value for an iterative algorithm in maximizing the function (Coelli, 1995b).

The loglikelihood function in terms of γ, as shown by Battese and Corra (1977), is given as

$$\ln(L) = -\frac{N}{2}\ln(\pi/2) - \frac{N}{2}\log(\sigma_s^2) + \sum_{i=1}^{N}\ln[1-\phi(z_i)] - \frac{1}{2\sigma_s^2}\sum_{i=1}^{N}\varepsilon_i^2 \tag{2.7}$$

where $z_i = \frac{\varepsilon}{\sigma_s}\sqrt{\frac{\gamma}{1-\gamma}}$ and all other notations are defined as before. The maximum likelihood (ML) estimates of β, σ^2 and γ are obtained by finding the maximum value of this loglikelihood function (Coelli et al., 1998).

2.3.2 Efficiency Estimates and Distribution of u_i's

The estimated parameters of the stochastic frontier model above are typically of secondary importance; of primary importance is the estimate of the inefficiency of firms, u (Greene, 2000). However, because u_i's are not observable, they cannot be estimated separately, neither can they be directly disentangled from v_i's in the composite error term ϵ_i ($=v_i-u_i$).[13] Instead, they are inferred by invoking the explicit formula for the expected value of u_i conditional on the composite error term, $E(u_i|v_i-u_i)$ (Jondrow et al., 1982). The required computation presupposes knowledge of the distributions of both v_i and u_i. While the consensus in the literature is that the random component v_i is independently and identically distributed with a normal distribution $N(0, \sigma_v^2)$, the selection of a particular distribution for the inefficiency component u_i cannot be justified *a priori*, as noted above.

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[13] This composite error term is for the stochastic production frontier. For the stochastic cost frontier, the error term would be $\epsilon_i=v_i+u_i$.

Thus, to estimate the inefficiency of farms, several distributions for the inefficiency error term u_i have been proposed in the literature, and used in empirical work.

The distributions that have been proposed include the half-normal (Aigner et al., 1977), exponential (Meeusen & van den Broeck, 1977), truncated-normal (Stevenson, 1980) and gamma (Greene, 1990). The half-normal $N^+(0, \sigma_v^2)$ and exponential $Ex(\theta)$ are relatively simple one-parameter distributions, rendering them easy to use, while the other two are more general distributions, with the potential advantage of providing flexibility with respect to the distributions of the inefficiency error term (Greene, 2000; Rossi & Canay, 2001). The truncated-normal $N^+(\mu, \sigma_v^2)$ distribution is a generalization of the half-normal such that its mean (or mode) μ is allowed to be non-zero, whereas the gamma is an extension of the exponential distribution. The gamma distribution, however, has been used with limited success because of the complexity of its loglikelihood function (Greene, 2000). Ritter and Simar (1997) point out that the procedure for maximizing the loglikelihood function (Greene, 1990) for the gamma distribution is not sufficiently accurate and that even an accurate estimator would still suffer from identification problems[14]. Therefore, this study investigates the use of the three distributions (half-normal, exponential, and truncated normal) that have been used successfully in empirical work.

Graphical representations of the half-normal, exponential, and truncated-normal distributions, for selected values of the parameters[15], are provided in Figures 2.2 to 2.4. The exponential distribution of u_i is characterized by the standard deviation parameter, σ_u, on whose value the probability density depends:

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[14] Greene (2000) acknowledges the problem of estimating his model and proposes an alternative approach based on the method of simulated maximum likelihood estimation, as contrasted to direct maximization of the loglikelihood function that was criticized by Ritter and Simar (1997). This new approach has not yet been used.

[15] The values used for the graphs are the same as those in Kumbhakar and Lovell (2000).

$$f(u)=\frac{1}{\sigma_u}\cdot\exp\left\{-\frac{u}{\sigma_u}\right\},\ u\geq 0 \tag{2.8}$$

Figure 2.2 shows exponential distributions corresponding to three different values of the standard deviation parameter, σ_u (=0.2. 0.5 and 1), showing their probability density near zero decreasing with increasing variance. The half-normal distribution also depends on σ_u,

$$f(u)=\frac{2}{\sqrt{2\pi}\sigma_u}\cdot\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\},\ u\geq 0 \tag{2.9}$$

The half-normal distributions shown in Figure 2.3 correspond to similar values of σ_u (=0.2. 0.5 and 1) as in Figure 2.2. As is the case for the exponential distributions, the proportion of probability density near zero of the half-normal distributions increases with decreasing variance. This has the economic implication that the majority of firms are almost efficient (Rossi & Canay, 2001).

The truncated-normal distribution, in contrast to the distributions discussed above, depends on two parameters σ_u and μ, where the latter is a placement parameter (mean or mode). Its density function is given by

$$f(u)=\frac{1}{\sqrt{2\pi}\sigma_u\Phi(-\mu/\sigma_u)}\cdot\exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2}\right\},\ u\geq 0 \tag{2.10}$$

This is the density function of a normally distributed random variable with either a zero or a non-zero mean μ, truncated at zero. Figure 2.4 shows two truncated distributions for a zero value and a positive value of μ when σ_u is set to unity. The distribution with the higher density in Figure 2.4 has a shape exactly similar to one half-normal distribution in Figure 2.3. This truncated distribution corresponds to the value μ=0, which, given σ_u, reduces to the half-normal density function (i.e., the one in Fig. 2.3 for which σ_u=0.2).

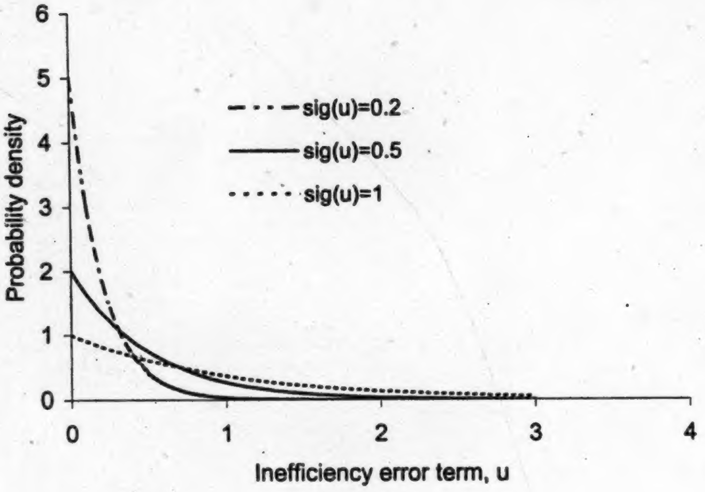

Figure 2.2 Exponential Distribution of u

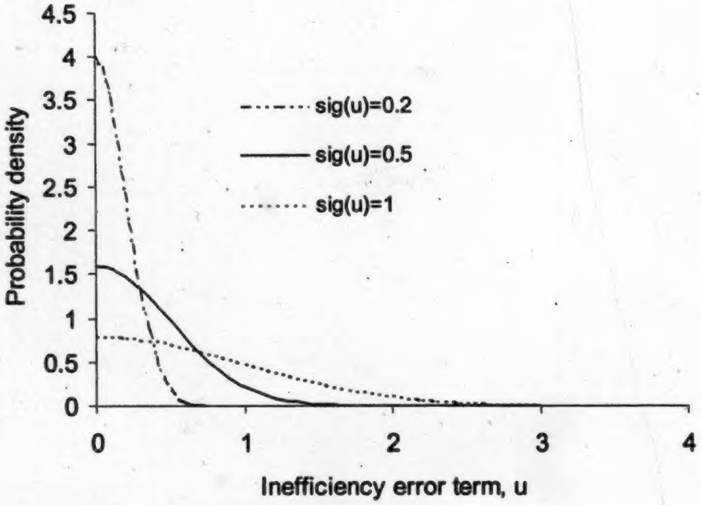

Figure 2.3 Half-Normal Distribution of u

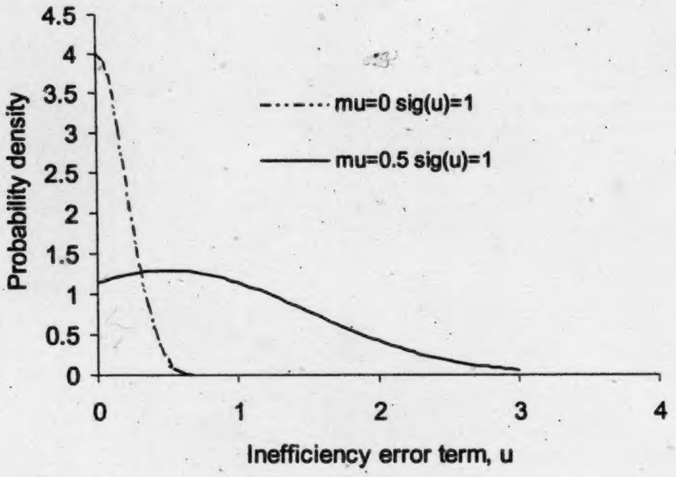

Figure 2.4 Truncated-Normal Distribution of u

However with $\mu > 0$, given σ_u, the probability density near zero decreases and the distribution assumes a longer tail, implying that the truncated-normal distribution will rank fewer firms as almost efficient than is the case for the half-normal distribution.

The point estimates of technical (economic) inefficiency for stochastic frontiers are obtained from the conditional distributions of u_i given the total composite error, ϵ_i (Jondrow et al., 1982).

$$f(u \mid \varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)} \tag{2.11}$$

The marginal density functions of ε, $f(\varepsilon)$, are derived as the marginal density functions of the joint density functions of ϵ and u as follows: (Kumbhakar & Lovell, 2000).

1) Exponential Distribution

For the Exponential Distribution, the following distributional assumptions are invoked

a) $v_i \sim$ iid $N(0, \sigma_v^2)$

b) $u_i \sim$ iid exponential

c) v_i and u_i are distributed independently of each other and of the regressors

The joint density function of u and v, $f(u,v)$, is the product of their individual density functions $f(u)$ (Eqn. 2.8) and

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} \cdot \exp\left\{-\frac{u^2}{2\sigma_v^2}\right\} \tag{2.12}$$

which, given the independence assumption of u and v, is given by

$$f(u,v) = \frac{1}{\sqrt{2\pi}\sigma_u\sigma_v} \cdot \exp\left\{-\frac{u}{\sigma_u} - \frac{v^2}{2\sigma_v^2}\right\} \tag{2.13}$$

From $\varepsilon = v - u$ (production frontier), v is expressed in terms u and ε as

$$v = \varepsilon + u, \tag{2.14}$$

Hence, $$f(u,\varepsilon)=\frac{1}{\sqrt{2\pi}\sigma_u\sigma_v}\cdot\exp\left\{-\frac{u}{\sigma_u}-\frac{1}{2\sigma_v^2}(u+\varepsilon)^2\right\} \qquad (2.15)$$

Thus, the marginal density function of ε for the exponential distribution is

$$f(\varepsilon)=\int_0^\infty f(u,\varepsilon)du=\left(\frac{1}{\sigma_u}\right)\cdot\Phi\left(-\frac{\varepsilon}{\sigma_v}-\frac{\sigma_v}{\sigma_u}\right)\cdot\exp\left\{\frac{\varepsilon}{\sigma_u}+\frac{\sigma_v^2}{2\sigma_u^2}\right\} \qquad (2.16)$$

where $\sigma=(\sigma_u^2+\sigma_v^2)^{1/2}$, $\lambda=\sigma_u/\sigma_v$,[16] and $\Phi(\cdot)$ and $\phi(\cdot)$ are, respectively, the standard normal cumulative distribution and density functions. Hence, for stochastic frontiers, the shape of the exponential distribution is determined by the standard deviation parameters, σ_u and σ_v.

2) Half-Normal Distribution

a) and b) The assumptions on v_i as iid normal and on the independence of u_i and v_i are the same as for the exponential distribution

c) u_i ~ iid $N^+(0,\ \sigma_u^2)$; that is, non-negative half normal.

Given the independence of *u* and *v*, the joint density function is given by the product of their individual density functions *f(u)* (Eqn. 2.9) and f(v) (Eqn. 2.12)

$$f(u,v)=\frac{2}{2\pi\sigma_u\sigma_v}\cdot\left\{-\frac{u^2}{2\sigma_u^2}-\frac{v^2}{2\sigma_v^2}\right\} \qquad (2.17)$$

Therefore,

$$f(u,\varepsilon)=\frac{2}{2\pi\sigma_u\sigma_v}\cdot\exp\left\{-\frac{u^2}{2\sigma_u^2}-\frac{(\varepsilon+u)^2}{2\sigma_v^2}\right\} \qquad (2.18)$$

Thus, the marginal density function of ε for the half-normal is given by

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[16] As Kumbhakar and Lovell (2000) explain, λ indicates the relative contribution of *u* and *v* to ε. As λ → 0 either $\sigma_v^2 \to +\infty$ or $\sigma_u^2 \to 0$, and the symmetric error component dominates the one-sided error component in the determination of ε. This indicates the case of OLS frontier function (without efficiency effects). Moreover, as λ → +∞ either $\sigma_u^2 \to +\infty$ or $\sigma_v^2 \to 0$, and the one-sided error component dominates the symmetric error component in the determination of ε. This indicates the case of the deterministic frontier (with no noise).

$$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)du = \frac{2}{\sqrt{2\pi}\sigma} \cdot \left[1 - \Phi\left(\frac{\varepsilon\lambda}{\sigma}\right)\right] \cdot \exp\left\{-\frac{\varepsilon^2}{2\sigma^2}\right\} \tag{2.19}$$

where $\sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}$, $\lambda = \sigma_u / \sigma_v$, and $\Phi(\cdot)$ and $\phi(\cdot)$ are, respectively, the standard normal cumulative distribution and density functions. Hence, the same standard deviation parameters σ_u and σ_v determine the shape of the half-normal distribution, as in the case of the exponential model.

3) Truncated-Normal Distribution

Since the truncated-normal model allows for a non-zero mode, it contains an additional parameter, μ, for its mode (mean).

The density *f(v)* of the truncated normal is the same as that of the half normal distribution (Eqn. 2.12) and the density function *f(u)*, for *u≥0*, is given by equation 2.10. Thus the joint density function of *u* and *v*, given their independence, is given by

$$f(u,v) = \frac{1}{2\pi\sigma_u\sigma_v\Phi(-u/\sigma_u)} \cdot \exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right\} \tag{2.20}$$

and the joint density of *u* and ε is given by

$$f(u,\varepsilon) = \frac{1}{2\pi\sigma_u\sigma_v\Phi(-u/\sigma_u)} \cdot \exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2} - \frac{(\varepsilon+u)^2}{2\sigma_v^2}\right\} \tag{2.21}$$

Hence, the marginal density of ε is given by

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma\Phi(-\mu/\sigma_u)} \cdot \Phi\left(\frac{\mu}{\sigma\lambda} - \frac{\varepsilon\lambda}{\sigma}\right) \cdot \exp\left\{-\frac{(\varepsilon+\mu)^2}{2\sigma^2}\right\}$$

$$= \frac{1}{\sigma} \cdot \phi\left(\frac{\varepsilon+u}{\sigma}\right) \cdot \Phi\left(\frac{\mu}{\sigma\lambda} - \frac{\varepsilon\lambda}{\sigma}\right) \cdot \left[\Phi\left(-\frac{\mu}{\sigma_u}\right)\right]^{-1} \tag{2.22}$$

where σ, λ, Φ and ϕ(·) are defined as before. Thus, in addition to the standard deviation parameters, σ_u and σ_v, the truncated-normal distribution for the stochastic frontier has a placement parameter, μ, that signifies the difference between the truncated-normal and half-normal marginal density functions. If μ=0, its marginal density function reduces to the half normal marginal density function.

2.3.3 Cost Frontiers (Measuring TE and AE)

The Aigner et al. (1977) model has a logical extension; specifically, an offshoot adapted for stochastic cost frontiers. Schmidt and Lovell (1979) developed this model by specifying a single-equation stochastic cost frontier of the form

$$\ln c_i = \ln c(q_i, w_i) + v_i + u_i \tag{2.23}$$

where c_i is the observed cost for firm i (i=1, . . .,N); q_i is a vector of output; w_i is a vector of input prices for firm i; u_i is a one-sided error term (i.e., positive for cost frontiers) capturing the effects of inefficiency; v_i is a two-sided random error accounting for variation in costs due to stochastic factors; $c(q_i, w_i)$ is the deterministic part of the cost equation; and $c(q_i, w_i)e^{v_i}$ is the stochastic cost frontier. Economic efficiency, which is estimated along with the stochastic cost frontier is defined for the i^th^ firm as,

$$EE_i = \frac{\exp(x_i\beta + u_i)}{\exp(x_i\beta)} = \exp(u_i) \tag{2.24}$$

where x_i is a vector of explanatory variables, including input prices and output.

Schmidt and Lovell (1979) specified a Cobb-Douglas technology for steam electric generating plants and demonstrated that a researcher could estimate the single-equation cost function using either maximum likelihood (ML) or corrected ordinary least squares (COLS) techniques in a manner similar to the estimation of stochastic production frontiers. In addition, they suggested the use of ML systems involving the cost function and K-1 factor share equations, K being the number of inputs. These systems provide more efficient estimators than the single equation estimators (Bauer, 1990; Coelli, 1995a).

The cost system that Schmidt and Lovell suggested is given by the following equations:

$$\ln c_i = \ln c_i(q_i, w_i) + \ln T_i + \ln A_i + v_i \tag{2.25}$$

$$s_{ij} = s_j(q_i, w_i) + e_{ij} \text{ for } j = 1,2,..k-1 \tag{2.26}$$

where c_i is the observed cost; $c(q_i,w_i)$ is the deterministic minimum cost frontier; q_i is a vector of outputs; w_i is a vector of input prices; $u_i = lnT_i + lnA_i$; lnT_i is a non-negative term that reflects the increase in cost due to technical inefficiency; lnA_i is a nonnegative term reflecting the increase in cost due to allocative inefficiency; v_i represent random errors; $s_j(.)$ is the observed cost share for the jth input; e_{ij} are the random errors in the jth input share equation; and K is the number of inputs.

This system has a number of relevant characteristics (Bauer, 1990). First, technical and allocative inefficiencies in the cost equation (2.25) increase observed cost, and therefore are one-sided. The firm will be allocatively inefficient by over or under-utilizing the inputs. The random errors v_i, however, can be positive or negative, implying that they could either increase or reduce the cost of production. Secondly, e_{ij}, a combination of random errors and allocative inefficiency in the share equation (2.26), may increase or decrease a given input's cost share, a result of the former's stochastic nature. Also, technical inefficiency does not appear in the input share equations because output is assumed to be exogenously determined. Thirdly, since both allocative inefficiency $\ln A_i$ in the cost equation and e_{ij} in the factor share equations amount to the added cost of inefficiency, they are functionally related. While $\ln A_i$ is the total inefficiency related to the misallocation of the inputs, each e_{ij} has a component of allocative inefficiency that is accountable to a particular input, which may be negative or positive depending on whether the input is under- or over-utilised relative to the cost-efficient input demands.

2.3.4 Cost System Approach and the "Greene" Problem

The "Greene" problem lies in analysing the error terms of the cost system (Greene, 1980), introduced in the previous section. Specifically, it relates to determining the most appropriate approach to model the relationship between lnA_i and e_{ij}, the terms that incorporate allocative inefficiency in the cost function and input share equations. While in the cost function the allocative inefficiency is one-sided, in the input share equations the error term (composed, in part, of allocative inefficiency) is two-sided. The relationship has been modelled using three alternative approaches (Bauer, 1990). The first is to rigorously derive the analytical dependence between the allocative inefficiency terms, e_{ij} and $\ln A_i$ (e.g., Schmidt & Lovell, 1979; Kumbhakar, 1988). The second approach is to assume, as Greene (1980) has done, that the allocative inefficiency error term in the cost equation is independent of the error terms in the share equations. The final approach is to model the relationship between all inefficiencies using an approximating function, imposing an assumed structure (Schmidt, 1984).

Using the analytic solution limits one to examining only self-dual functional forms (e.g., Cobb-Douglas technology). Schmidt and Lovell (1979) used the Cobb-Douglas production function to derive the system of cost and factor demand equations and to functionally map the error terms in the factor demand equations into the allocative inefficiency error term in the cost equation[17]. They solve the "Greene" problem through functionally mapping the disturbances in the factor demand equations into the allocative inefficiency term in the cost equation. This relationship is expressed as $\ln A = E - \ln r$, where E is a function of the Cobb-Douglas production function coefficients and the error term on the factor demand equations and *r* is the returns to scale (RTS)[18]. Technical inefficiency is given as $\ln T = -u/r$, a function of the returns to scale and the one-sided disturbance in the production frontier.

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[17] Kumbhakar (1988) generalized Schmidt and Lovell's approach to allow for multiple outputs and fixed inputs.

Because the analytic techniques depend on using self-dual functional forms, they cannot help to solve the "Greene" problem if a more flexible functional form is specified. Using a translog cost system (with a Gamma distribution for the cost inefficiency error term) Greene (1980) attempted to deal with the problem by ignoring the link among allocative inefficiency error terms across the equations. The error terms in the share equations were assumed to be statistically independent of the inefficiency term in the cost equation and the former were assumed to have a multivariate normal distribution with mean zero. Greene's approach, by ignoring the relationship among the allocative inefficiency error terms, is not fully efficient statistically. Nevertheless, it does not necessarily yield worse results than an approach that models the relationship incorrectly (Bauer, 1990).

Schmidt (1984) developed the approximating method by proposing a specification that ensures a) $lnA_i = 0$ when $e_i = 0$ and b) lnA_i and e_{ij} are positively correlated for all j. This approach has been criticized as modelling the relationship incorrectly (Bauer, 1990). Moreover, because the technique yields a complicated likelihood function, it has never been used to obtain empirical estimates. Melfi (1984) simplified Schmidt's (1984) specification and obtained a more tractable likelihood procedure by making no assumptions about the distribution of lnA and assuming no cross correlation among the input share equations, so that $\ln A_i$ is the sum of the squared errors on all the share equations. Bauer (1985) extended Melfi's approach to a more flexible estimation technique.

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[18] For a C-D function, $q = a\prod_{j=1}^{M} x_j^{\alpha_j} e^{\varepsilon}$, $E = [\sum_{m=2}^{M}(\alpha_m / r)\varepsilon_m + \ln[\alpha_1 + \sum_{m=2}^{M}\alpha_m e^{-\varepsilon_m})]$ and $r = \sum_{m=1}^{M}\alpha_m$.

The approximation techniques still have two drawbacks. First, the techniques require researchers to estimate a large number of parameters, even in cases where there are a small number of outputs and inputs. Second, use of this approach does not necessarily lead to better estimates of the cost frontier (Bauer, 1990)[19].

In the current study, a single equation approach for modelling economic inefficiency is adopted, as suggested by Coelli (1995a). This approach has the limitation of ignoring information contained in the system of equations (i.e., cost plus factor shares). Hence, using a single-equation approach rather than a system-of-equations estimation is at the expense of sacrificing the opportunity of obtaining more asymptotically efficient estimates of efficiency and technology. This is offset, however, by the advantage of not having to estimate a complex system of equations and address the "Greene" problem. In other words, by estimating a single equation (i.e., the cost function), the "Greene" problem is circumvented in this study.

2.4 Empirical Efficiency Studies of the Dairy Sector: Canada and US

This section reviews empirical studies of technical and/or economic efficiency for dairy farms in Canada and the U.S. Discussion of the efficiency studies from these countries is combined because these countries compete for the Northern American market of milk and other dairy products. The reviewed studies point to differences in issues such as definitions of efficiency, methods used to estimate any single type of efficiency, and choice of functional form or the structure of the error term (Richards & Jeffrey, 2000).

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[19] In addition to Bauer (1990), which forms the basis for the discussion in this section, Greene (1993) also provides information relevant to this problem.

Despite the wide spectrum of efficiency analysis, some issues related to efficiency have not been given sufficient attention, which the present study attempts to address.

2.4.1 Canadian Studies

Weersink et al. (1990) used cross-sectional data for Ontario dairy farms to compute and decompose TE measures into purely technical, congestion and scale efficiency measures, using a deterministic non-parametric programming approach. Pure technical inefficiency results from producing within the isoquant frontier; congestion inefficiency is due to overutilization of inputs (i.e., MPP< 0); and scale inefficiency is due to deviations from constant returns to scale. Therefore, pure technical efficiency refers to producing on the isoquant frontier and scale efficiency refers to producing at the optimal scale of production from a technical perspective (i.e., being on the right isoquant). In addition to this decomposition, Weersink et al. (1990) used a censored regression to examine factors that influence technical efficiency.

They found overall TE to range from 0.65 to 1.00, with an average of 0.92. The major sources of inefficiency were pure technical inefficiency (0.95 TE on average) and non-optimal scale of production (0.97 on average). Congestion efficiency did not show any significant effect. TE tended to increase with the herd size, milk yield, and butterfat content of milk, and to decrease with greater levels of purchased feed (measured as a proportion of the total feed input) and over-capitalisation (as indicated by the amount of debt to total assets and too much machinery and barn capacity).

Romain and Lambert (1992) used 1990 data from Ontario and Quebec sample farms to estimate a deterministic production frontier (among others). This frontier was then used for several purposes; to analyse cost of production according to farm size and yield per cow; to determine the level of TE for dairy farms; to analyse the relationship between cost of production (COP) and TE; and to investigate factors that influence TE.

Their results suggested that relative to the least efficient farms in each province, the costs of production for the most efficient farms are 16% lower in Quebec and 13% lower in Ontario. For both Quebec and Ontario and for different herd size categories, increased labour productivity and decreased costs are associated with higher levels of TE, although they do not indicate significant economies of size. Labour productivity increases with herd size for all levels of TE. When the level of TE is considered, cash costs and costs of production are not affected by the size of the farm. Efficient farms have lower costs of production, a result of their efficient use of production factors. Lastly, factors that characterise efficient farms, as shown by covariance analysis, include level of education, participation in milk-recording program, quality of forage, and the year in which the manager joined a management club; that is, there is high correlation between efficient farms and these variables.

Cloutier and Rowley (1993) used cross-sectional data (1988 & 1989) from Quebec dairy farms to estimate a deterministic non-parametric milk production frontier, using DEA methods. This frontier was used to assess the relative technical efficiency of Quebec dairy farms, compare efficiency levels between the two years, and check the sensitivity of the scores with respect to herd size and sample size. They found higher technical efficiency scores for 1989 (0.91vis a vis 0.88 for 1988). Furthermore, efficiency scores were higher for small herds than for large ones when examined in the overall estimation. However efficiency scores for the large herds were greater than for small ones when the herds were categorised into small (28-39 head) and large (40-60 head) and separate estimations of efficiency levels conducted. They concluded that the DEA procedure seems to be unduly sensitive to the size of the sample, which requires more research to establish the robustness of individual ranking for scores.

Richards and Jeffrey (1996) used pooled cross sectional data from 1989 to 1991 to estimate a parametric stochastic production frontier and a cost equation for Alberta dairy farmers. A Cobb-Douglas form was specified and estimated for the production frontier. As well, a quadratic cost equation was estimated using the AF (iterative OLS) and composite error (CE) ML method. TE was computed from the stochastic production frontier and, using OLS techniques, was regressed on variables that would explain its variation.

For both the CE and AF methods milk output was positively related to levels of grain and concentrates, hired labour, and family and operator labour. Hay and forage was also found to exert positive influence on output with the AF method. With respect to the cost equation, result for the AF method suggested that the variables positively related to costs included hay, TE, producer age and square of the number of cows. The CE method showed milk yield and the square of milk yield to have a positive influence and herd size and TE to have a negative effect. The estimates for average TE were almost identical between the two methods (i.e., 0.85 for the AF method versus 0.83 for the CE method). Lastly, technical efficiency was found to increase with milk yield per cow and capital to labour ratio, and to decrease with producer age.

Richards and Jeffrey (2000) used the same data set (1989 -1991) to estimate a stochastic parametric cost frontier of the Cobb-Douglas form. They further decomposed economic efficiency into TE and AE using a method developed by Kopp and Diewert, (1982). Taking economic efficiency as a measure of performance (a latent variable), they examined the factors that explain economic performance of farms using the latent variable model (multiple cause, multiple indicator). The model has one endogenous variable (performance), five exogenous variables (herd size, square of herd size, yield per cow, square of yield per cow, and producer's age) and three variables (breed, feed, and labour) that are used as "proxies" for the unobservable measures; specifically, the quality of the producer's breeding programme, feeding programme, and labour inputs, respectively.

The results indicated an average TE of 0.94, average AE of 0.96 and average EE of 0.91. TE and AE, (and hence EE) appear to be very similar among herd size groups. The maximum economic efficiency herd size in the sample is 70 cows, and maximum economic efficiency yield occurs at 90.3 hectolitres per cow per year. Constant Returns to Scale (CRS) could not be rejected in the sample. The study concluded that performance might improve by improving the quality of the breeding programme and labour productivity. Experience did not exert a significant influence on efficiency.

Mbaga et al. (2000) used Quebec dairy data (categorized between the maize and non-maize producing regions) to estimate stochastic production frontiers and to measure and assess the robustness of technical efficiency in milk production in relation to the assumed distribution of the inefficiency error term. In addition, they used the results on TE measures as a gauge for choosing a dominant functional form among the Cobb-Douglas, translog and generalized Leontief formulations. The parametric TE results were then compared with the non-parametric (DEA) results.

The results showed the following. 1) The average technical efficiency measures were generally high. For example, in the maize region, all TE estimates are higher than 91%. For the dominant form, the average TE ranges between 0.95 and 0.97. 2) The use of the dominance criterion showed that the generalized Leontief functional form dominated the Cobb-Douglas and the translog functions. 3) The TE measures from different functional specifications and across the assumed distributions of the inefficiency term (half-normal, truncated and exponential) were all high (rank correlation coefficients >0.947; correlation coefficients >0.92). From these results, Mbaga et al. (2000) concluded that the functional form and the assumed distribution for the inefficiency error term were not critical in assessing the ranking of the level of technical efficiency of dairy farms. 4) The TE measures from parametric estimations were not comparable to those obtained from DEA as indicated by relatively low correlation and rank correlation coefficients (correlation coefficients < 0.5; rank correlation coefficients < 0.6). 5) Results for output elasticities suggest that for concentrate, forage and labour, the elasticities decrease input use in both regions; in the maize region, the elasticity with respect to capital increases with input use, which indicates underutilization of capital in that region.

2.4.2 U.S.A. Studies

Bravo-Ureta (1986) estimated the TE of dairy farms in the New England region of the United States using a deterministic Cobb-Douglas frontier production function. The parameters of the production function were estimated using linear programming methods involving the probabilistic frontier approach (Timmer, 1971). Technical efficiency levels ranged from 0.58 to 1.00, with an average of 0.82. A Chi-square test was performed to determine whether technical efficiency of individual farms and farm size (measured by the number of cows) were statistically independent. The hypothesis that TE and farm size are independent was not rejected at 1 percent level of significance.

Tauer and Belbase (1987) estimated technical efficiencies for a sample of New York dairy farms using 1984 cross sectional data fitted to a deterministic log-linear Cobb-Douglas function, estimated by COLS. In addition, they regressed the estimated TE values on variables hypothesized to influence technical efficiency. These were dummy variables to measure differences in technology (i.e., type of barn and location within the state and herd size (i.e., number of cows)), proxy variables for management inputs (age and education), and type of record keeping (e.g., mail-in record keeping system), among others.

The findings showed that the average farm was 69 percent technically efficient. Factors leading to greater technical efficiency were found to include favourable location (i.e., regions with the most productive soils and best weather in the state) and larger herd size. Participation in the Dairy Herd Improvement Cooperative (DHIC) and use of mail-in computerised records resulted in reduced efficiency. However, their model explained only 9 percent of the variation in efficiency.

Bravo-Ureta and Rieger (1990) estimated and compared results of deterministic and stochastic frontier production functions using data for 1982 and 1983 from a sample of dairy farms in the northeastern states of the U.S.A. The parameters of the deterministic model were estimated using linear programming methods (Aigner & Chu, 1968), maximum likelihood methods on the assumption that the non-negative farm effects had a gamma distribution (Greene, 1980), and COLS (Richmond, 1974). Parameters of the stochastic frontier were estimated by ML techniques, assuming the farm effects had a half-normal distribution (ALS).

The results showed the existence of technical inefficiency for the stochastic frontier model for 1982, which was not significantly different from the frontier in 1983. Moreover, although the estimated technical efficiency of farms from the three methods used for the deterministic model showed considerable variability, the values were generally lower than those obtained by the use of the stochastic frontier. However, the technical efficiencies by these methods were found to be highly correlated, and gave a similar ranking of farms.

Khumbhakar et al. (1989) applied a stochastic production frontier to estimate technical, allocative and scale efficiencies for a sample of Utah dairy farms. This stochastic production frontier, which included both endogenous and exogenous variables, was estimated as a system together with the first order condition equations from profit maximisation (with inefficiency). The endogenous variables included labour (both family and hired labour) and capital (the opportunity cost of capital expenses on the farm), while the exogenous variables included level of formal education, off-farm income and measures of farm size.

The results showed the following: both endogenous and exogenous variables were positively related to farm size, large farms were more efficient than other types of farms, there was a positive relationship between years of farming and productivity of capital and labour, and a negative relationship between off-farm income and productivity.

Khumbhakar et al. (1991) used a generalised parametric production frontier model to examine the profitability of US dairy farms using 1985 cross sectional data from 28 states. They examined profitability in relation to the following issues: returns to scale (RTS), the impact of AE and TE on profit levels for farms of different sizes, and a test of whether the Cobb-Douglas frontier function fitted the data. In addition, factors used to explain the technical efficiency of farms included education, region and farm size. All of the parameters of the models were estimated jointly by ML procedures.

The results of their study indicated that a Cobb-Douglas function was not appropriate. Elasticities of output with respect to inputs were lower for large farms. Education should be included in the production frontier as an input that affects production through improving TE rather than as a conventional exogenous input in the production function. RTS was less than unity, with mean values of 0.945, 0.904, and 0.544 for small, medium and large farms, respectively. In addition, large farms were more technically and allocatively efficient than smaller farms. Khumbhakar et al. (1991) argued that since profitability is inversely related to RTS, the findings above indicated that large farms are more profitable relative to small and medium sized farms.

Tauer (1993) estimated deterministic, non-parametric production and cost frontiers for a sample of New York farms, using 1990 cross sectional data. These frontiers were used to calculate TE and AE and to compare the efficiency of farms in the short run (SR) and long run (LR). The models were estimated using linear programming methods, assuming variable returns to scale (VRS). A separate estimation identified factors that explained economic efficiency by estimating a logistic function using Ordinary Least Squares (OLS).[20]

The results showed that whereas average TE was higher in the LR than in the SR (0.79 versus 0.74), the converse was true for EE (0.70 versus 0.87). There was a positive relationship between TE and AE. Milk production was shown to exhibit increasing returns to scale in the LR.

Thomas and Tauer (1993) applied a deterministic, non-parametric stochastic production frontier to examine the effect of linear aggregation of inputs by value on TE. Inputs, as given by the cash expense, were grouped into 28 categories. Different levels of aggregation were used for similar inputs. Using linear programming and assuming constant returns to scale, they showed that with increased linear aggregation, the number of efficient farms falls as does the value of TE measures. In addition, linear aggregation was shown to affect the efficiency ranking of individual firms. They concluded that the source of this downward bias is allocative inefficiency, which makes the measured efficiencies a combination of technical and allocative efficiencies.

Ahmad and Bravo-Ureta (1995) used panel data (1971-1984) from a sample of Vermont dairy farms to decompose milk output growth into technological change, technical efficiency change and input growth using a parametric stochastic production frontier and the fixed-effects frontier. In their estimations, they applied the ML techniques from the Battese and Coelli (1992) model for the stochastic frontier and the fixed-effects model. They found that i) mean efficiency was 0.77, with slight variation for the whole sample, ii) size played a greater role than productivity growth in increasing milk output, and iii) the role of technical efficiency in productivity growth was minor. The major contribution to total output growth came from technological progress.

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[20] Tauer (1993) exploited the fact that the computed inefficiencies are bounded numerically between zero and one and fitted the logistic function, $E_i = 1/(1+e^{-(\alpha+\beta X_i)})$. This function is bounded by an open set (0,1), where E_i is the computed inefficiency, e is the numerical constant, X is a vector of explanatory variables, and α and β (vectors) are the estimated coefficients. Rearranging and taking logs of both sides resulted in the linear estimated function $\ln(E_i/1-E_i))=\alpha+\beta X_i$. Since all inefficiencies were greater than zero, they were all shifted downwards by 0.001so that the largest inefficiency was 0.999 rather than 1.0, which would have produced $\ln(\infty)$. Regressions were then estimated using OLS.

Ahmad and Bravo-Ureta (1996) used the same data set to examine the impact of fixed effects vis a vis stochastic frontiers on the technical efficiency. Applying the ML techniques [illegible] the Battese and Coelli (1992) model of the stochastic frontier and for the fixed-effects model, they fitted the Cobb-Douglas and simplified translog specifications to the panel data. They found the production of milk to exhibit increasing returns to scale and the estimated technical efficiency measures to be invariant as to the Cobb-Douglas and translog specifications.

2.4.3 Summary

Studies in the foregoing review addressed the following issues related to technical and economic efficiency:

1) Estimation of frontiers:

This includes parametric and non-parametric approaches of estimating frontiers, and within these approaches, stochastic and deterministic methods. The frontiers are estimated with a view to measuring efficiency. While technical efficiency is measured from estimated production frontiers, economic efficiency is measured from estimation of either the cost or the profit frontiers. Some studies have decomposed economic efficiency into technical and allocative efficiency.

2) Examination of factors that explain efficiency:

Efficiency analysis is conducted in either of two ways: by estimating the inefficiency model simultaneously with the frontier, or by using two stages in which the efficiencies are first computed from the frontiers and then regressed on variables that explain their variation. Variables in the inefficiency models include some variables used in the production frontiers as well as latent variables or dummy variables, the selection of which is not predicated on any theoretical underpinnings. Typically, the choice of the variables to include in the model is justified by common reasoning. These variables attempt to capture the effect of technology, farm size, breeding, feeding, education, farming experience, location, as well as other management characteristics.

3) Deriving or testing theoretical and/or methodological issues (assumptions) related to estimated frontiers:

The main issues addressed in this regard include computing return to scale (RTS), output elasticity of inputs and input price elasticities; comparing and testing the efficacy of functional forms, comparing methods of estimation, and examining economies of scale and profitability of farms in relation to efficiency levels.

In analysing these issues, the findings of the reviewed studies have varied, as have the approaches and methods used. As Richards and Jeffrey (2000) point out,

> There are many ways to estimate any single type of efficiency and... within each method, [even] one's choice of functional form or the structure of the error can cause different efficiency measures or efficiency rankings for a single observation [among other things]. p. 233

Very few of the reviewed studies, for example, have tested alternative functional forms; many have relied on single period cross sectional data. Others have confined themselves to examining only technical efficiency, without addressing either economic or allocative efficiency; or examining factors that explain efficiency. Romain and Lambert (1992) contend that the costs of dairy production can decrease in two ways. One is with improved management of inputs, which calls for an understanding of the factors that explain technical inefficiency. Another is better choice of inputs in the production process according to their relative costs, which calls for an understanding of whether or not the farms are utilising the inputs fully and allocating them optimally.

The conflicting results highlighted above point to the need for further research to address some of the issues that have not been resolved or given sufficient attention to date (at least in North American studies). In this study, both production and cost frontiers are estimated using data from Alberta dairy farmers. Technical and economic efficiencies are calculated from the estimated frontiers, and then analysed. In addition, the efficacy of different functional specifications is statistically tested and a number of factors that explain the efficiency of farms examined. Unlike most of the reviewed studies, which used cross sectional data, this study used panel data, thereby allows an examination of efficiency over time.

2.5 Empirical Studies Examining Alternative Distributions of the Inefficiency Error Term

In empirical research, the choice of which distribution to use or the effects of adopting alternative distributions have not been given sufficient attention. The focus has been primarily on stochastic frontiers, with inefficiency of firms estimated by assuming a particular distribution (the preference being the half-normal). The question that is yet to be answered satisfactorily is this: Does the assumption adopted for the distribution of inefficiency matter in terms of the resulting efficiency estimates? A few studies that have examined this empirical question have not converged at the same answer. Whereas efficiency measures have been found to be sensitive to the distribution assumption adopted, it is still not so clear as to whether the ranking of producers by their efficiency scores is also sensitive to the assumed distribution (Kumbhakar & Lovell, 2000). Also still inconclusive is the sensitivity of the efficiency estimates to the choice of error distribution among alternative functional specifications (Mbaga et al., 2000).

The empirical results from using alternative distribution assumptions in the previous studies have been analysed in three ways. One way has been to compare mean efficiency estimates resulting from the same data, with most of the results confined to the half-normal and exponential distributions. Another has been to rank efficiency estimates from either the same data or different data. Lastly, efficiency estimates have been ranked across alternative functional specifications.

The mean efficiencies from empirical work that used both the half-normal and exponential distributions indicate that the exponential distribution tends to identify a larger number of efficient firms than the half-normal. Rossi and Canay (2001) used four databases from previous studies[21] to test the performance of the two distributions. Their results conform to the above assertion, with the mean efficiencies for the exponential distribution being higher than those for the half-normal distribution. A similar conclusion was arrived at for mean TE in some studies that estimated production frontiers using ML (Jaforullah & Devilin, 1996; Parikh & Shah, 1996; Mbaga et al., 2000[22]) and for mean EE in those that estimated cost frontiers by ML (Greene, 1990[23]; Parikh & Ali, 1995). However, Jaforullah (1996) obtained mixed results with regard to mean TE for a set of companies of Bangladesh, though in general the exponential distribution resulted in higher mean TE than did the half-normal.

Ranking efficiency estimates with respect to a number of distributions gives results that are not as homogeneous as those with respect to mean efficiency. Mbaga et al. (2000), using Quebec dairy data, computed both the level and rank correlation coefficients of individual technical efficiency estimates resulting from the truncated-normal, the half-normal and the exponential distributions of inefficiency. The efficiency estimates were not only highly correlated (0.92 - 0.99) among the three distributions for each particular functional specification but were also so among the alternative functional forms of production technology considered in the study (i.e., Cobb-Douglas, translog and Generalised Leontief). Rossi and Canay (2001) also found very high rank correlations (0.959 - 0.993) for the rankings of efficiency estimates obtained by ML methods. Kumbhakar and Lovell (2000) used Greene's (1990) results to calculate the rank correlation coefficients of individual economic efficiency estimates related to several distributions of inefficiency and obtained results that ranged between 0.75 (exponential and gamma) and 0.98 (half-normal and truncated-normal). On the other extreme, Giannakas et al. (1998) obtained very low rank correlation coefficients of technical efficiency estimates across alternative specifications.

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[21] The data were previously used in Rossi (2000), Estache and Rossi (1999), Stewart (1993) and CEER (2000). By estimating both production and cost frontiers in diverse sectors (gas, water and electricity), Rossi and Canay (2001) aimed at making the results robust (Rossi & Canay, 2001).

[22] Mbaga et al. (2000) compared mean TE for three distributions; namely, the exponential, half-normal and truncated distributions. Estimations were carried out for two sets of data. For both data sets the exponential distribution model was found to result in the highest mean TE and the half-normal distribution the least. However, as the authors note, the difference is very marginal.

[23] Greene (1990) estimated a stochastic cost frontier for a cross-section of 123 U.S. electrical utilities and obtained the following results for sample mean efficiencies: 0.8766 (half normal), 0.9011 (exponential), 0.8961 (truncated normal) and 0.8949 (gamma).

This short review raises three issues for further examination. The first one is the choice between assuming a relatively simple distribution (i.e., exponential or half-normal) versus a more flexible distribution (e.g., truncated-normal), as well as the choice between different relatively simple distributions. Empirical evidence tends to suggest that the choice of the distribution would influence the level of efficiency estimates and that these estimates would be sensitive to changes in the distribution. Ritter and Simar (1997) have suggested the use of relatively simple distributions rather than a more flexible distribution (such as the truncated-normal). This suggestion would call for choosing only between the half-normal and exponential distributions. However, their suggestion is based on the analysis of the normal-gamma distribution rather than on both the truncated-normal and gamma distributions. Moreover, available empirical evidence has indicated a very high correlation between the results of the half-normal and truncated-normal distributions. These two distributions are sufficiently similar (the former being nested in the latter) as to deserve further empirical examination. Thus, more evidence with regard to the choice between the relatively simple distributions and the more flexible truncated-normal may be helpful in validating the suggestion by Ritter and Simar (1997).

Secondly, the examination of all three of the alternative distributions discussed may be important for yet another reason. As Mbaga et al. (2000) have concluded, the robustness of efficiency estimates to various distributions of the error term is specific to the data used. This may imply that the choice should not be generalized from the literature; rather, it should be based on empirical analysis of the data used. With the exception of Mbaga et al. (2000), previous North American studies in the dairy sector have not addressed this issue (to the best of the knowledge of this author). Whereas Mbaga et al. (2000) analysed it in relation to only technical efficiency, this study examines it with respect to not only technical efficiency but also economic efficiency.

The third issue is the effect of the distribution assumptions on the choice of functional form for the stochastic frontiers. To what extent, for example, are the efficiency results related to a particular distribution assumption consistent among alternative functional specifications? Ordinarily, the choice of functional form between alternatives is carried out for estimates related to a specific distribution of the inefficiency error term. Implicit in this procedure is the assumption that the maintaining of the same distribution of the inefficiency error term has no significant effect on the level as well as the ranking of the resulting efficiency estimates for alternative functional specifications. However, as evidence from some studies has indicated, the results of the efficiency estimates (for the same distribution) may turn out to display weak correlation between alternative functional specifications.

Whether this is solely a problem of a particular study or is a general problem in certain respects is a question that further evidence may serve to enlighten. Otherwise, if the implicit assumption behind choosing between alternative functional specifications does not hold, then it may be difficult to decide in favour of one of the alternative forms without having tested how the chosen functional specification would fare under alternative assumptions of the distribution of the inefficiency error term.

Chapter 3. Empirical Methods

This chapter elaborates on methodological issues as well as empirical models and data used in the estimation. It is divided into two sections. Section 3.1 examines technical and economic efficiency within the frameworks of stochastic production and cost frontiers, respectively. Coupled with this analysis is the discussion of functional forms and time aspects in the formulation of stochastic frontier empirical models for the study. As well, the section discusses the models for estimating efficiency of farms for alternative distribution assumptions of the inefficiency error term. Section 3.2 reports on the sources and construction of data used in this study. This includes a discussion on the variables for the production frontiers, the cost frontiers and the inefficiency models.

3.1 Stochastic Frontier Analysis (SFA)

3.1.1 Examining Technical Inefficiency

In this sub-section, technical inefficiency is considered as part of the total error term for the stochastic production frontier. Stochastic frontier analysis (SFA) is used to separate technical inefficiency from the error attributable to random factors. The process entails estimating stochastic production frontiers and technical efficiency of farms.

For these estimations, the Battese and Coelli (1995) model is used. This model extends further the framework for estimating the stochastic production frontiers and technical inefficiency independently proposed by Aigner et al. (1977) and Meeusen and van den Broeck (1977). The original works did not show, for example, how one could predict technical efficiencies for individual sample firms. However Jondrow et al. (1982) resolved this shortcoming by presenting two predictors for technical efficiency effects of firms, assuming the parameters of the production frontier were known and data (cross sectional) were provided. Waldman (1984) investigated the properties of a predictor for firm effects proposed by Jondrow et al (1982).

The Battese-Coelli (1995) model was appealing for the present task because it generalizes some of the results of the studies mentioned earlier (i.e., Chapter 2). First, it accommodates unbalanced panel data in sample firms; second, it allows technical efficiency of firms to vary over time and it includes a time variable to capture technical change; and third, it assumes that Stephenson's (1980) general distribution of firm effects applies for the stochastic frontier production function[24]. A discussion of functional forms and specification of the stochastic production frontier model for Alberta dairy farmers is presented in the next section (Section 3.1.2). In general, this stochastic frontier is given as,

$$q_{it} = f(GC_{it}, HF_{it}, L_{it}, K_{it}, OI_{it}, YR_{it}; \beta)\exp(V_{it} - U_{it}) \qquad (3.1)$$

where, for the ith farm in year t,

q_{it} =quantity of milk output in hectolitres per year per cow.

GC_{it}=quantity of grains and concentrates, as tonnes per year per cow.

HF_{it} = quantity of hay and forage, as tonnes per year per cow.

L_{it} = quantity of labour (operator, hired and family labour), as average hours per year per cow; computed from the total average hours per year as measured by the total wage bill divided by the average wage rate.

K_{it} = capital (valued using constant 1992 prices) per cow per annum.

OI_{it} = real expenditure on other inputs (i.e., utilities, breeding and veterinary expense, etc), in constant 1992 prices in $ per cow.

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[24] Stevenson (1980) specified for a stochastic frontier a truncated normal distribution for technical inefficiency effects, u_i, to address the criticisms that the half-normal or exponential distributions are arbitrary selections. Coelli et al. (1998) point out that both the half-normal and exponential distributions have a mode at zero, which implies relatively high technical efficiency. The truncated normal distribution is a generalisation of the half normal distribution, obtained by the truncation at zero of the normal distribution with mean, μ, and variance, σ^2.

YR_t = the year of observation ($t = 1, \ldots, N$).

V_{it}-U_{it}= combined error term

β is a vector of parameters to be estimated.

The explanatory variables in the above production frontier model constitute factor inputs that influence the amount of milk produced per cow. Thus, greater usage of any input should lead to increased milk output, which would be indicated by a positive relationship between the dependent variables and the explanatory variables[25].

The sum of GC and HF constitutes the total amount of feed for the herd. Although in some studies the two types of feed are aggregated as one variable (e.g., Moschini, 1988), these two types of feed contain different nutritional content. Therefore, they are likely to contribute differently to the output of milk, for which reason they were treated as separate variables.

The time trend variable, YR, permits neutral or non-neutral technical change, depending on how it is specified. Technical change is a concept used to define comparisons of productivity through time. It involves advances in technology, which may be represented by an upward shift in the production frontier. Neutral technical change affects inputs in the same way so that there is no change in relative input use, i.e. input proportions stay the same. With non-neutral technical change the inputs are affected differently; therefore productivity change varies across inputs. Neutral change is represented as the effect of the rate of growth of output on the "aggregate input", that may be measured by the coefficient(s) of the time variable, with the effect of interaction of the time variable and input variables being zero. The non-zero effect(s) of the latter represents non-neutral technical change for flexible forms (e.g., the conventional translog form discussed in the next section), whereas for the Cobb-Douglas form, non-neutral technical change can be represented by shifts in the production elasticities.[26]

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[25] This presupposes that the farmers are not operating within stage three of the production process.

The V_{it}'s are assumed to be **iid** random errors having $N(0, \sigma_v^2)$ distribution, and the U_{it}'s are **iid** nonnegative random variables, representing the effect of technical inefficiency of the farms involved. In the Battese-Coelli (1995) model, these U_{it} variables are obtained by a truncation (at zero) of an **iid** normal distribution with unknown mean, μ, and unknown variance, σ^2. The variance of the parameters is given as

$$\sigma_s^2 = \sigma_v^2 + \sigma^2 \tag{3.2}$$

$$\gamma = \sigma^2 / \sigma_s^2 \tag{3.3}$$

where the γ parameter takes on values between zero and one.

The technical inefficiency latent model is given by:

$$U_{it} = Z_{it}\delta + r_{it} \tag{3.4}$$

where Z_{it} is a (1xM) vector of explanatory variables associated with the technical inefficiency effects, δ is an (Mx1) vector of unknown parameters to be estimated, and the r_{it}'s are unobservable random variables, which are assumed to be independently distributed, obtained by truncation of the normal distribution with mean zero and unknown variance, σ^2, such that U_{it} is non-negative (i.e., $r_{it} \geq - Z_{it}\delta$).

Specifically,

$$U_{it} = \delta_0 + \delta_1(YF_{it}) + \delta_2(H_{it}) + \delta_3(KL_{it}) + \delta_4(GH_{it}) + \delta_5(BE_{it}) + \delta_6(YR_i) + r_{it} \tag{3.5}$$

where, for the i^{th} farm in year t,

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[26] Note that there is a difference between the technological change in the production frontier framework (the primal rate of technological change) and in the cost frontier (the dual rate of technological change). The production frontier shifts upward with technological change, and cost frontier shifts downward, i.e., since more output is produced for a given input x, the total cost of producing any given output rate is lower. The rate of the technological change in the cost frontier framework is the product of the primal rate of the technological change and the elasticity of cost with respect to output ($\partial \ln C / \partial \ln Q$) (see Carlson et al. 1993 for derivations). Thus the primal and dual rates of technological change are equal if and only if $\partial \ln C / \partial \ln Q$ is unity; i.e., if and only if the technology exhibits constant returns to scale.

YF_{it} = years of farming (a proxy for farming experience),

H_{it} = herd size,

KL_{it} = capital-to-labor ratio,

GH_{it} = ratio of grain and concentrates expense to hay and forage expense,

BE_{it} = Breeding and veterinary expense per cow,

YR_{i}= Time trend variable

r_{it}= error term

The variables used to statistically explain technical inefficiency relate to those in Richards and Jeffrey (1998), who explained economic efficiency of Alberta dairy farms using a latent variable model.[27] Included also in the model for technical efficiency is a time trend variable, YR, which Richards and Jeffrey did not include because their data spanned a short time period. In contrast to the production frontier model, YR in the technical efficiency model captures temporal changes in efficiency against the shifting frontier. Other variables explain the role of genetic advancement and sophistication in dairy breeding (BE), effect of variation in feed quality (GH), and effect of improvements in dairy milking and feeding technology (KL) (Richards & Jeffrey, 1998).

The inefficiency model (3.5) includes some of the same explanatory variables as are in the stochastic frontier model (3.1). This is justified analytically because they affect both models and empirically because the inefficiency is assumed to be stochastic (Battese & Coelli, 1995; Coelli et al. 1998).

The random r-variables r_{it} are defined by the truncation of the normal distribution (with zero mean and variance σ_r^2) such that the point of truncation is $-Z_{it}\,\delta$, (which implies that $r_{it} \geq - Z_{it}\delta \ \forall \ i, t$). The variables are independently but not necessarily identically distributed neither are they necessarily non-negative; and the mean $Z_{it}\delta$ is not required to be positive for each observation (Battese & Coelli, 1995). Thus, the technical efficiency of production for the i^{th} firm at the t^{th} observation is defined by:[28]

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[27] An adjustment to Richards and Jeffrey (1998) model was considered, in which the ratio of the number of milk cows to total livestock (CH) was included in addition to (or in place of) herd size. The rationale was that more calves add to cost of producing milk in terms of the inputs they consume but at the same time imply a lower percentage of dry cows in the herd, hence more milk per cow. The ratio was hypothesized also to have a bearing on the culling process. Thus, CH variable was assumed to influence the level of efficiency. Parameter estimates for the CH variable were statistically insignificant. In addition, including both H and CH variables affected sign and significance of H in some models. The variable CH was therefore dropped from the final models and herd size was retained.

$$\begin{aligned} TE_{it} &= \exp(-U_{it}) \\ &= \exp(-Z_{it}\delta - r_{it}) \end{aligned} \tag{3.6}$$

The mean technical efficiency for the whole sample was computed as a simple average of individual farm efficiency. To obtain this mean, technical efficiency of farms was gauged on the production of best performing farms; that is, farms for which output is located on the estimated production frontier. Average technical inefficiency for the whole sample is the proportion of output by which the "average" producer falls short of full technical efficiency. This is measured as the difference between full and mean efficiency; that is, a proportion of output not realized by the farms, on average, because the inputs that went into producing its output were not fully utilized.

Moreover, the technical inefficiency model was estimated by regressing technical efficiency on a set of variables that were hypothesized to affect it. The time trend was used in this model to examine whether or not there was any statistically significant change in efficiency over time.

In equations 3.4 and 3.5, a positive sign for an estimated δ coefficient implies that the associated variable has a negative effect on efficiency, and vice versa. For each explanatory variable (in the technical inefficiency model), there was *a priori* expectation concerning the "sign" of the coefficient, as summarized below.

- ***Farming Experience***

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[28] This definition is correct only when the dependent variable is in logarithms, which is the case for the translog functional form. As well, the Cobb-Douglas form is typically specified in logarithmic terms when empirically estimated.

Higher levels of farming experience are hypothesized to be associated with lower levels of inefficiency (i.e., a negative sign for the parameter estimate). This is based on the supposition that farmers will learn from their mistakes and improve on their production with time, leading over time to a reduction in technical inefficiency.

- ***Herd Size***

Although evidence as regards to the influence of herd size is not empirically conclusive, larger herd sizes were expected to be associated with lower levels of inefficiency since large farms would potentially reap advantages associated with economies of scale (i.e., a negative sign for the parameter estimate).

- ***Capital-to-Labour Ratio***

Capital intensity is expected to enhance efficiency. Hence, a negative sign is expected for the coefficient on capital to labour ratio.

- ***Breeding and Veterinary Expense***

These were expected to have a negative influence on the degree of inefficiency (i.e., negative sign): more of the expense would lead to healthier animals, enabling them to produce more milk. Increased cost due to breeding expense may be a result of better genetics (i.e., better quality semen), a technological effect. By improving the quality of some of the inputs (e.g., the efficiency of feed), increased breeding expense may lead to more milk output, with the indirect effect of enhancing technical efficiency. Veterinary cost, however, may be incurred to solve health problems and lower animal productivity, leading to more milk output, which is a direct effect of enhancing technical efficiency.

- ***Grain and Concentrates-to-Hay and Forage Ratio***

The effect of nutrition upon the degree of inefficiency could be positive or negative. This is explained at length in Foley et al. (1972). They argue that the amount of concentrates that should be fed to the cow depends on her level of milk production and the energy value of the concentrate mix being fed.

> If in the herd all cows are fed the same amount of concentrates, the low producers will be overfed and get fat, and the high producers will be underfed and lose weight. The high producers may be able to maintain a high level of milk production for a short time by using their body fat reserves as a source of energy; but when they are depleted, milk production will decrease. (Foley et al., 1972, p. 254)

Moreover, they point out that the optimal quantities of grain and roughage to be fed depend upon the genetic potential for milk production of the cows, the quality of the roughage, the cost of the dairy ration or the supply of home grown grain, and the price of milk. For maximum profits in this respect, Foley et al. (1972) contend that the amount of grain fed should be increased until the cost of last unit of concentrates just equals the value of the extra milk produced.

- ***Time Trend, YR***

A negative coefficient on the YR variable was expected because the process of learning and adopting a technology is perfected over time. This may lead to reduced levels of technical inefficiency.

3.1.2 Estimation Procedures

The FRONTIER 4.1 program (Coelli, 1994; 1996) was used to simultaneously estimate, by ML estimators[29], the parameters for both the stochastic frontier and the model explaining technical inefficiency effects; that is, the β- and δ-coefficients together with the variance parameters of equations (3.2) and (3.3).[30]

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[29] The ML estimator is asymptotically more efficient than the COLS estimator (Coelli, et al., 1998). In addition, Coelli (1995b) investigated the finite-sample properties of the half normal frontier model in a Monte Carlo experiment, in which the ML estimator was found to be significantly better than the COLS estimator when the contribution of the technical inefficiency to the total variance term is large. Therefore, he suggested that, wherever possible, ML estimators should be used instead of COLS estimators.

Earlier studies (e.g., Pitt & Lee, 1981; Kalirajan, 1981; Kalirajan, 1991) adopted a two-stage approach. In the first stage, a researcher specifies and estimates the stochastic production frontier and predicts the technical inefficiency effects, under the assumption that these inefficiency effects are identically distributed. In the second stage, a regression model for the predicted technical inefficiency effects is specified and estimated. Kalirajan (1991) defends this approach by claiming that farm-specific factors exert only indirect influence on production through their association with inefficiency.

This two-stage approach has been criticized on two grounds. First, using inefficiency effects as a dependent variable in the second stage contradicts the assumption of identically distributed efficiency effects in the stochastic frontier (Coelli, 1995a). In fact, if inefficiency is correlated with the inputs, the estimates of both the production frontier and the inefficiency will be inconsistent (Kumbhakar et al., 1991). Second, as the authors argue, it is inappropriate to use the inefficiency index as a dependent variable in OLS regression because it is one-sided.[31]

3.1.3 Functional Forms

Functional forms are specific to both model and data. However, most efficiency studies focus solely on determining the degree of inefficiency and do not examine alternative specifications of the technology. However, if researchers choose a form that is incorrect, their model will potentially predict responses in a biased and inaccurate way (Griffin et al., 1987). The consequences of this error may include, among others, misleading policy implications (Giannakas et al., 1998).

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[30] FRONTIER is a computer programme, written by Coelli, to provide maximum likelihood estimates of parameters for a variety of stochastic production and cost functions. While the programme is very accommodating in what it can do, it has limitations. For example, it cannot accommodate exponential or gamma distributions, nor can it estimate systems of equations. For details about FRONTIER, see Coelli (1994, 1996).

[31] Some studies have addressed this problem by transforming the dependent variable using the logistic function (e.g., Tauer, 1993; Amara et al., 1999).

In this study, the Cobb-Douglas and the transcendental logarithmic (translog) functional forms were specified and estimated. In addition to being the most commonly used functional forms, these forms allow comparisons to be made between the findings of the current study relative to previous studies that have analysed Alberta dairy efficiency. For example, Richards and Jeffrey (1996) used the translog form; Richards and Jeffrey (1998) used the C-D form.

The Cobb-Douglas (C-D) frontier production function is specified in logarithmic form, as follows:

$$\ln q_{it} = \beta_0 + \sum_{j=1}^{4} \beta_j \ln x_{jit} + \beta_t t \ + (v_{it} - u_{it}) \tag{3.7}$$

where, for the j^{th} input and the i^{th} firm in time period t, q is output, x are inputs, v are random errors, u are technical efficiency effects, and β's are parameters to be estimated.

The Cobb-Douglas form is used mainly because of its simplicity and parsimony (Richards & Jeffrey, 1998). Also, because the C-D production function is self-dual, the corresponding cost frontier can be derived analytically (see Varian, 1992, Chapter 4). Moreover, by transforming the model into logarithms, one obtains a model that is linear in inputs and thus is straightforward to estimate (Coelli, 1995a). Some studies justify using the C-D form by referring to Kopp and Smith's (1980) conclusion that the functional form has a limited effect on empirical efficiency measurement. The C-D form, although useful, has several limitations. The elasticity of substitution between any pair of inputs for the C-D form is restricted to unity. The C-D form is also restrictive with respect to returns to scale, which take the same value across all firms in the sample and are constant across output levels. The C-D form is also inflexible in that it provides only a first order approximation to a function, which limits its ability to approximate other functions. As well, it assumes that all inputs are technical complements.

The trans-logarithmic (or translog) function is specified as

$$\ln q_{it} = \beta_0 + \sum_{j=1}^{J} \beta_j \ln x_{jit} + \beta_t t + \frac{1}{2}\beta_{tt} t^2 + \sum_{j=1}^{J} \beta_{jt} \ln x_{jit} t + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{K} \beta_{jk} \ln x_{jit} \ln x_{kit} + v_{it} - u_{it} \tag{3.8}$$

where all variables and parameters are defined as above.

The translog is the most commonly used flexible functional form, in that it can provide a second order approximation to an arbitrary twice differentiable linearly homogeneous function (Diewert, 1976). Hence, the translog does not impose restrictions on the structure of the technology such as restrictions on returns to scale or elasticity of substitution. It permits the elasticity of substitution to be determined by the data (Chambers, 1988). The main drawbacks associated with the translog are its susceptibility to multicollinearity and the potential problem of insufficient degrees of freedom due to the presence of interaction terms (Coelli, 1995a).

The Cobb-Douglas and translog are among the functional forms nested in the generalized quadratic Box-Cox (GQBC) functional form, written as

$$q_{it}^{(\lambda_1)} = \beta_0 + \beta_t t + \beta_{tt} t^2 + \sum_{j=1}^{J} \beta_j x_{jit}^{(\lambda_2)} + \sum_{j=1}^{J} \beta_{jt} x_{jit}^{(\lambda_2)} + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{K} \beta_{jk} x_{jit}^{(\lambda_2)} x_{kit}^{(\lambda_2)} + v_{it} - u_{it}\,,$$

i=1,2,...,N and *t*=1,2,...,T (3.9)

where *j* and *k* stand for inputs used in producing output, *i* represents farms and *t* is time. The β's, λ_1 and λ_2 are the parameters to be estimated; ε_{it} is the random error; $q_{it}^{(\lambda_1)}$ and $x_{it}^{(\lambda_2)}$ are the Box-Cox transformations, defined as:

$$q_{it}^{(\lambda_1)} = \frac{q_{it}^{\lambda_1} - 1}{\lambda_1} \text{ and } x_{it}^{(\lambda_2)} = \frac{x_{it}^{\lambda_2} - 1}{\lambda_2} \tag{3.10}$$

This form has been used previously in frontier analysis to examine the relative performance of different functional forms (Zhu et al. 1995, Giannakas et al. 1998). The translog and C-D forms result from applying appropriate restrictions on the values of λ_1 and λ_2 of the GQBC. The GQBC becomes a translog form when $\lambda_1=0$ and $\lambda_2=0$. It becomes the Cobb-Douglas form when in addition to $\lambda_1=\lambda_2=0$, the second order parameters (β_{jk}) are assumed to equal zero for all j,k.[32] However, the GQBC may sometimes fail to select one of the examined forms. Furthermore, Unterschultz and Mumey (1996) caution that GQBC estimations are prone to problems and biases caused by heteroscedasticity and/or autocorrelation as well as by data scaling, and that these problems can seriously bias the transformation variable and invalidate statistical tests. Thus, the GQBC was not used in the empirical part of the study.

The present study focuses on two functional forms, whereby the C-D form is considered as a special case of the translog frontier (Battese & Broca, 1996). The C-D form is derived from the translog form by restricting the coefficients of the second order terms of the translog to zero, that is, $\beta_{jk}=0$ for all $j \leq k$.

3.1.4 Time Aspects

Researchers have modelled inefficiency as either time varying or time-invariant. Time-invariant models are suitable for short panels. Since the panel data set for the present study is sufficiently long, the time varying approach has been adopted. In empirical studies, this approach has been modelled in three ways.

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[32] For a discussion concerning the imposition of these restrictions and the resulting functional forms, see Giannakas et al. (1998).

One method is to allow only the error component representing technical inefficiency to be time varying (Kumbhakar, 1990; Battese & Coelli, 1992)[33]. A second method involves modelling inefficiency through the intercept of the production frontier such that each producer has its own intercept, and the intercept is allowed to vary quadratically through time at producer-specific rates (Cornwell et al., 1990).[34] A third method is to allow the error component to be time varying and also to include a trend variable among the regressors (Battese & Coelli, 1995; Heshimati & Kumbhakar, 1995; Heshimati et al., 1995). This is captured in the trend variable (YR) in the stochastic frontier and specified in FRONTIER 4.1.

The third method is used in this study primarily because it lends itself to clear interpretation. Whereas the trend variable in the frontier production function is associated with technical change, the error component U_{it}, associated with technical efficiency, is allowed to vary across producers and through time. Moreover, the first two methods suffer from a number of inadequacies. With the first, every producer has the same pattern of technical efficiency change; in addition, it does not explain the behavioural or institutional motivation for the time variance of technical efficiency. The second provides two differing interpretations that cannot be distinguished empirically: either the parameters for inefficiency represent producer-specific levels of technical inefficiency, or u_{i1} represent the producer-specific initial (or persistent) levels of inefficiency and the u_i ($i = 2,3,\ldots,T$) represent producer-specific technical change (Lovell,

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[33] Battese and Coelli (1992), for example, specify $U_{it} = \{\exp[-\eta(t-T)]\}u_i$ where η is an unknown parameter to be estimated. This model, which is incorporated in FRONTIER, specifies that the technical inefficiency effects in earlier periods of the panel are a deterministic exponential function of the inefficiency effects in the last periods of the panel. Technical inefficiency effects are time-invariant if $\eta = 0$. The hypothesis for this is specified as $H_0: \eta = 0$ and tested using the likelihood ratio (LR) test.

[34] Cornwell et al. (1990) did this by generalizing Schmidt and Sickles' (1984) transformed model for time-invariant inefficiency effects. The Schmidt and Sickles (1984) model is specified as $y_{it}=\alpha_i+x_{it}\beta+v_{it}$ where $\alpha_i=\alpha+u_i$. Cornwell et al. replaced the firm effects, u_i, by $u_{it}=\theta_{i1}+\theta_{i2}t+\theta_{i3}t^2$, which allows the efficiency levels to vary over firms and time.

1996). Given the specifications of the time-varying model for the inefficiency effects, the null hypothesis of time-invariant effects was not tested, since as discussed in the next chapter, time did not have a significant bearing on the efficiency of Alberta farms.

3.1.5 Examining Economic Efficiency (EE)

Economic efficiency was calculated from the estimated stochastic cost frontiers. A two-step method was followed in the estimations. The first step was to estimate total economic efficiency by fitting a stochastic cost frontier in which the composite error term accounts for total economic inefficiency. Then, since economic efficiency is a composite product of technical and allocative efficiency, an attempt was made to decompose economic efficiency into its two components.

The methods used and the procedure followed to obtain economic inefficiency were the same as those used in the case of technical efficiency: both the Cobb-Douglas and translog cost frontiers were estimated, using the FRONTIER programme, from which economic efficiency was computed. The reported economic efficiency is the simple average of efficiency measures of individual farms.

The stochastic cost frontier for Alberta dairy farms is given as

$$c_{it} = f(Q_{it}, PGC_{it}, PHF_{it}, PL_{it}, PK_{it}, POC_{it}, YR_i; \beta_t)\exp(V_{it} + U_{it}) \tag{3.11}$$

where for the ith farm in year t,

c_{it}= Total cost in \$ per year per cow

Q_{it}=Milk output in hectolitres per year per cow

PGC_{it}= Average price of grain and concentrates per tonne in \$

PHF_{it}= Average price of hay and forage per tonne in \$.

PL_{it}= Average (wage rate) price of labour per hour in \$, an average of wage rates for operator and hired labour.

PK_{it}= per unit price of capital, as given by the user cost of capital.

POC_{it}=Price of other costs (for all the remaining intermediary inputs), as represented by the deflator of the input price index (1992=100).

YR_i= the year of observation (in terms of 1, . . . , N)

U_{it}= one-sided error term (i.e. positive for cost frontiers)

V_{it}= a two-sided random error accounting for variation in costs due to stochastic factors

β_t are parameters to be estimated.

The economic inefficiency latent model, as in the case of technical inefficiency, is given by:

$$U_{it} = Z_{it}\xi + n_{it} \qquad (3.12)$$

where Z_{it} is a (1xM) vector of explanatory variables associated with the economic inefficiency effects, ξ is an (Mx1) vector of unknown parameters to be estimated, and the n_{it}'s are unobservable random variables, which are assumed to be independently distributed, obtained by truncation of the normal distribution with mean zero and unknown variance, σ^2, such that U_{it} is non-negative (i.e., $n_{it} \geq - Z_{it}\xi$).

Specifically,

$$U_{it} = \xi_0 + \xi_1(YF_{it}) + \xi_2(H_{it}) + \xi_3(KL_{it}) + \xi_4(GH_{it}) + \xi_5(BE_{it}) + \xi_6(YR_i) + r_{it} \qquad (3.13)$$

where the parameters and explanatory variables are defined the same as those discussed earlier (i.e., section 3.1.1).

The Cobb-Douglas cost function is specified in logarithmic form as

$$\ln c_{it} = \beta_0 + \beta_1 \ln q_{it} + \sum_{j=2}^{m} \beta_j \ln w_{ijt} + \beta_t t + v_{it} + u_{it} \qquad (3.14)$$

whereas the translog cost function is specified as

$$\ln c_{it} = \alpha_0 + \alpha_y \ln q_{it} + \tfrac{1}{2}\alpha_{yy}(\ln q_{it})^2 + \sum_j \varphi_{yj}(\ln q_{it})(\ln w_{ijt}) + \sum_j \alpha_j \ln w_{ijt} +$$
$$\tfrac{1}{2}\sum_j \sum_k \beta_{jk}(\ln w_{ijt})(\ln w_{ikt}) + \varphi_{yt}\ln(q_{it})t + \omega_t t + \tfrac{1}{2}\omega_{tt}t^2 + \tfrac{1}{2}\sum_j \omega_{jk}(\ln w_{ijt})t + v_{it} + u_{it} \tag{3.15}$$

where in both functions w_{ijt} are input prices, the other variables are as defined above, and the β's, α's, φ's and ω's are parameters to be estimated.

The direct estimation of the single cost function is at the expense of some gains in efficiency that would be realized by estimating a cost system that includes also the optimal, cost-minimizing input demand equations, which may be alternatively transformed into cost share equations (Eqn. 3.23). The joint estimation of the cost function with the corresponding share equations ordinarily facilitates the imposition of theoretical constraints that follow from the regularity conditions, described in Section 2.2.2.

For homogeneity in input prices, the following constraints are normally imposed on the parameters of the translog cost function: $\sum_{j=1}^{n}\alpha_j = 1$; $\sum_{k}^{n}\beta_{jk} = \sum_{k}^{n}\beta_{kj} = \sum_{j}^{n}\varphi_{qj} = 0$. The adding-up constraints of the share equations system require that the cost shares (Eqn. 3.23) sum to unity, following from $\sum_{i}^{n} w_i x_i = c$, which also implies that $\sum_{j=1}^{n}\alpha_j = 1$. In this study, homogeneity was alternatively imposed by normalizing input prices and total cost relative to the price of labour. Moreover, the adding-up condition was not directly imposed, since only the single cost equation was estimated, but the total cost data for each farm was calculated as a sum of input cost shares. In addition to these, symmetry restrictions were also imposed, i.e., $\beta_{jk} = \beta_{kj}\ \forall\, k \neq j$.

In the second step, overall economic efficiency was decomposed into its technical and allocative components, following the Kopp and Diewert (1982) method.

This two-step approach was suggested as a way to circumvent the "Greene problem" (Coelli, 1995a).

A variant of the Kopp-Diewert (1982) method, used also by Bravo-Ureta and Evenson (1994), is used to decompose overall economic efficiency into technical and allocative components by calculating three ratios utilizing three different estimations of cost of production. The alternative, used elsewhere, involves calculating the ratios of input vector norms (e.g., Kopp & Diewert, 1982; Richards & Jeffrey, 1998). The costs of production related to Kopp and Diewert (1982) efficiency measures are described below.

Actual cost is first computed using the actual input bundle $\mathbf{x}^{\mathbf{A}}$, obtained at observed input prices $\mathbf{w}^{\mathbf{A}}$ and used to produce the observed output **q**. This cost measure includes total economic inefficiency, entailing a combination of technical and allocative inefficiency. The second computation is optimal total cost, as computed at the means of output level and input prices from the estimated cost frontier. Since this is a measure of the minimum cost for producing a specified amount of milk, it reflects technical as well as allocative efficiency in production. The third computation is the estimate of the cost of producing milk output with a calculated bundle $\mathbf{x}^{\mathbf{B}}$ that is constructed such that output is produced efficiently in a technical sense, but inefficiently, in an allocative sense, relative to the optimal level. At the calculated cost minimizing input prices $\mathbf{w}^{\mathbf{B}}$, this bundle would lead to attaining also allocative efficiency in producing output q.

Kopp and Diewert (1982) start by obtaining the cost-minimizing input factor demands implied by Shephard's Lemma. Because the estimated frontier cost function is taken to represent a locus of economically efficient points, these input demand functions derive from the first partial derivatives of the estimated cost frontier with respect to actual input prices. The underlying implicit assumption is that the Shephard's Lemma supplies the technically and allocatively efficient input bundle **x**^{E}. With reference to Figure 3.1[35], efficiency measures were calculated as discussed below[36].

- ***Overall Economic Efficiency***

To assess the overall economic efficiency, the bundle **x**^{C} is calculated. By construction, bundle **x**^{C} represents input use in the same proportions as bundle **x**^{A}, the actual input bundle, but it costs the same as the technically and allocatively efficient bundle, **x**^{E}. The bundle **x**^{E} (corresponding to the coordinates of point E in Figure 3.1) is obtained by invoking Shephard's Lemma and differentiating the cost frontier with respect to each of the input prices.

$$x^E = \bar{v}_w c(q, w^A) \tag{3.16}$$

where $\bar{v}_w c(q, w^A) \equiv \left[\frac{\partial c}{\partial w_1^A}, \frac{\partial c}{\partial w_2^A}, \frac{\partial c}{\partial w_3^A}, \frac{\partial c}{\partial w_4^A}, \frac{\partial c}{\partial w_5^A} \right]$ and q>0 is the output produced by the inefficient point **x**^{A} and w^A is a vector of input prices for the used inputs at x^A. Hence, at point E the producer is *economically* efficient.

The input quantities for this economically efficient input vector, **x**^{E}, are obtained by substituting the farm's input prices and output quantity in the input (factor) demand system that is given by (3.16) above. The input vector **x**^{C}, which in Figure 3.1 is shown by the point of intersection with the iso-cost plane **ww'** of the line segment joining the origin to **x**^{A} is given as

$$x^C \equiv \lambda^C x^A \text{ where } \lambda^C = \frac{c(q, w^A)}{w^A \cdot x^A} \tag{3.17}$$

---

[35] Although Kopp and Dewert (1982) argued that their technique was only applicable to deterministic cost functions, Greene (1993) notes that if the cost functions were stochastic frontiers instead, the only difference in the computation would be the addition of an estimate of v_i to the cost function.

[36] This figure is the same as Figure 2.1. It is replicated here to facilitate the reference to it.

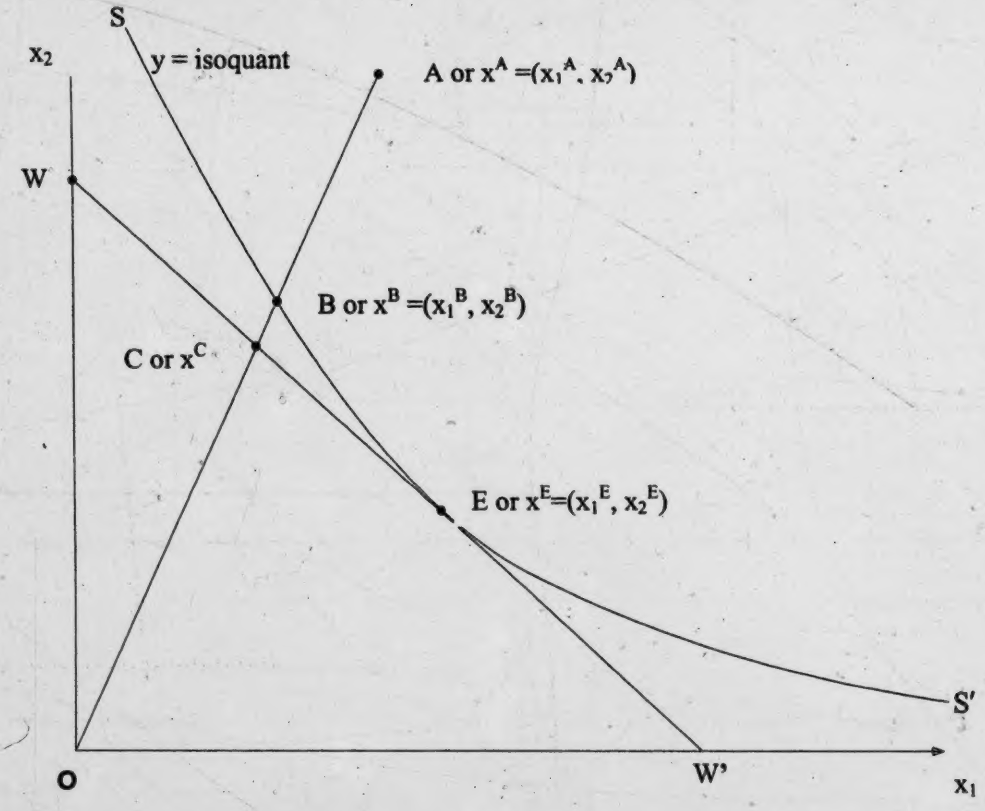
S
y = isoquant
x_2
A or $x^A = (x_1^A, x_2^A)$
W
B or $x^B = (x_1^B, x_2^B)$
C or x^C
E or $x^E = (x_1^E, x_2^E)$
S′
O
W'
x_1

Figure 3.1 Graphical Illustration of Efficiency Concepts

Overall economic efficiency is calculated numerically for average input usage as the ratio between the cost of producing the observed output with the efficient bundle $\mathbf{x}^E$ and that of producing it with the actual input bundle. In other words, it is the ratio between optimal cost of producing the average output (as derived using the estimated cost frontier) and the actual cost of producing it (as given by the average of farms' costs of production).

$$EE = \frac{w^A \cdot x^C}{w^A \cdot x^A} = \frac{c(q, w^A)}{w^A . x^A} \qquad (3.18)$$

- ***Technical Efficiency (TE) and Allocative Efficiency (AE)***

To decompose overall economic efficiency into technical and allocative components the bundle $\mathbf{x}^B$ was required. This is a bundle that lies along the same isoquant depicting the observed output q, which implies that point B is technically, though not allocatively, efficient. Second, by construction, the input proportions at $\mathbf{x}^A$ are the same as at $\mathbf{x}^B$. Kopp and Diewert observe that there is some vector of prices of inputs, $\mathbf{w}^B$, for which $\mathbf{x}^B$ is also allocatively efficient.

$$x^B = \nabla_w c(q, w^B) \qquad (3.19)$$

for some set of input prices $\mathbf{w}^B >> 0$. The issue is to derive simultaneously the relative prices $\mathbf{w}^B$ and the input quantities constituting $\mathbf{x}^B$. This is achieved by invoking the following equalities.

1) Equality of the factor input proportions of $\mathbf{x}^B$ and $\mathbf{x}^A$:

$$\frac{x_i^A}{x_N^A} = \frac{x_i^B}{x_N^B} \qquad \text{i=1,. .,5} \qquad (3.20)$$

This equality in general gives N-1 equations, as the last one $x_N^B / x_N^B = x_N^A / x_N^A$ is an identity.

2) The demand system implied by Shephard's Lemma:

$$x_i^B = \frac{\partial c(q, w^B)}{\partial w_i^B} \quad i = 1, \ldots, N \tag{3.21}$$

By normalizing $w_N = 1$ (since only relative prices are relevant), one obtains N-1 unknowns in N equations,

$$x_i^B = \partial c(q, w_1^B \ldots, w_{N-1}^B) / \partial w_i \qquad i = 1, \ldots, N \tag{3.22}$$

Therefore, the system has 2N-1 equations in 2N-1 unknowns $x_1^B, \ldots x_5^B; w_1^B, \ldots, w_4^B$, (equations 3.19 and 3.20). With the number of equations equaling to the number of unknowns, the values of the $\mathbf{x}^B$ vector and a vector of relative prices $\mathbf{w}^B$ are solved simultaneously.

The above derivation was applied to the estimated translog cost frontier for Alberta farms' dairy milk output (equation 3.13). First, the equations for factor demands, i.e., $x_i = \frac{\partial C}{\partial w_i}$ were derived. Since the translog cost function above is expressed in logarithms, the factor demand functions are easily expressed as cost shares of inputs:

$$s_i = \frac{w_i x_i}{C} = \frac{\partial \ln C}{\partial \ln(w_i)} \Rightarrow x_i = \frac{C}{w_i} \cdot \frac{\partial C}{\partial w_i} \tag{3.23}$$

The cost of milk production for Alberta dairy farmers is the total sum of the product of five inputs [GC HF L K OI] = $[x_1\ x_2\ x_3\ x_4\ x_5]$ and the corresponding input prices, w^A_i, i=1,2,3,4,5. A vector of unknown input prices at point B, where production is technically efficient but allocatively inefficient, is given as w^B_i, i=1,2,3,4,5. Normalizing the price of "other inputs", $w^B_5=1$, results in four relative prices w^{B*}_i, i=1,2,3,4 and the unit cost of "other inputs". To obtain the input demand functions for Alberta dairy producers at point B, the estimated Cobb-Douglas and translog cost frontiers were differentiated with respect to w_i^B. For the translog form, these factor demand functions (depicted by point B in Figure 3.1) that would enable farms attain only technical efficiency are expressed by the following 5 equations:[37]

$$x_1^B = \frac{C}{w_1}(\delta_{y1} \ln q + \alpha_1 + \beta_{11} \ln w_1^B + \beta_{12} \ln w_2^B + \beta_{13} \ln w_3^B + \beta_{14} \ln w_4^B) \tag{3.24a}$$

$$x_2^B = \frac{C}{w_2}(\delta_{y2} \ln q + \alpha_2 + \beta_{12} \ln w_1^B + \beta_{22} \ln w_2^B + \beta_{23} \ln w_3^B + \beta_{24} \ln w_4^B) \tag{3.24b}$$

$$x_3^B = \frac{C}{w_3}(\delta_{y3} \ln q + \alpha_3 + \beta_{13} \ln w_3^B + \beta_{23} \ln w_2^B + \beta_{33} \ln w_3^B + \beta_{34} \ln w_4^B) \tag{3.24c}$$

$$x_4^B = \frac{C}{w_4}(\delta_{y4} \ln q + \alpha_{44} + \beta_{14} \ln w_1^B + \beta_{24} \ln w_2^B + \beta_{34} \ln w_3^B + \beta_{34} \ln w_4^B) \tag{3.24d}$$

$$x_5^B = \frac{C}{w_5}(\delta_{y5} \ln q + \beta_{15} \ln w_1^B + \beta_{25} \ln w_2^B + \beta_{35} \ln w_3^B + \beta_{45} \ln w_4^B) \tag{3.24e}$$

The ratios of x^B functions (equations 3.24a-e) are then equated to the ratios of the corresponding actual inputs (equation 3.20) i.e., $\frac{x_N^A}{x_i^A} = \frac{x_N^B}{x_i^B} = b_i$, giving a system of 4 equations expressed as functions of 4 unknowns, the **W** vector.

$$\frac{\delta_{y5} \ln q + \beta_{15} \ln w_1 + \beta_{25} \ln w_2 + \beta_{35} \ln w_3 + \beta_{45} \ln w_4}{\delta_{y1} \ln q + \alpha_{11} + \beta_{12} \ln w_1 + \beta_{12} \ln w_2 + \beta_{13} \ln w_3 + \beta_{14} \ln w_4} \cdot w_1 = b_1 \tag{3.25a}$$

$$\frac{\delta_{y5} \ln q + \beta_{15} \ln w_1 + \beta_{25} \ln w_2 + \beta_{35} \ln w_3 + \beta_{45} \ln w_4}{\delta_{y2} \ln q + \alpha_{22} + \beta_{12} \ln w_1 + \beta_{12} \ln w_2 + \beta_{23} \ln w_3 + \beta_{24} \ln w_4} \cdot w_2 = b_2 \tag{3.25b}$$

$$\frac{\delta_{y5} \ln q + \beta_{15} \ln w_1 + \beta_{25} \ln w_2 + \beta_3 \ln w_3 + \beta_{45} \ln w_4}{\delta_{y3} \ln q + \alpha_{33} + \beta_{13} \ln w_1 + \beta_{23} \ln w_2 + \beta_{33} \ln w_3 + \beta_{34} \ln w_4} \cdot w_3 = b_3 \tag{3.25c}$$

$$\frac{\delta_{y5} \ln q + \beta_{15} \ln w_1 + \beta_{25} \ln w_2 + \beta_{35} \ln w_3 + \beta_{45} \ln w_4}{\delta_{y4} \ln q + \alpha_{44} + \beta_{14} \ln w_1 + \beta_{24} \ln w_2 + \beta_{34} \ln w_3 + \beta_{44} \ln w_4} \cdot w_4 = b_4 \tag{3.25d}$$

The equation system for the Cobb-Douglas form was derived in the same way.

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[37] The coefficients of the factor demand function were restricted to take into account Young's theorem, i.e., $\beta_{ij} = \beta_{ji}$.

These systems of equations, however, could not be solved for w^B_i's.

TE and AE that are computed from the Kopp and Diewert's decomposed measures, which possess the same interpretation as the original Farrell measures, are defined as follows (Kopp & Diewert, 1982; Bravo-Ureta & Evenson, 1994):

- **TE** is equal to the ratio of the total cost of producing output q with input bundle $\mathbf{x}^B$ (at a vector of prices $\mathbf{w}^B$) to the actual total cost.

$$TE = \frac{w^B \cdot x^B}{w^A \cdot x^A} \tag{3.26}$$

- **AE** is equal to the ratio of the total cost of producing q with bundle $\mathbf{x}^C$ (at a vector of prices $\mathbf{w}^A$) to the total cost of producing it with bundle $\mathbf{x}^B$ (at a vector of prices $\mathbf{w}^B$).

$$AE = \frac{w^A . x^B}{w^B \cdot x^B} = \frac{c(q, w^A)}{w^B \cdot x^B} \tag{3.27}$$

Allocative efficiency was calculated only for the Cobb-Douglas formulation, based on the property that the function is self-dual, using the estimates of EE from the cost frontier and TE from the production frontier[38].

3.1.6 Estimating Efficiency In Relation to Various Distributions of u_i

This sub-section presents empirical methods that were used to analyse the sensitivity of efficiency estimates to the assumed distribution for u_i (including the exponential, half-normal and truncated-normal distributions). Drawing from previous studies, the analysis was gauged on the following hypotheses:

- In comparing the estimates from alternative distributions, the exponential distribution assumption will result in farms being ranked as more efficient than the half-normal (Rossi & Canay, 2001), with the half-normal distribution resulting in farms being more efficient than the truncated normal distribution.

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[38] In a strict sense, self-duality of the Cobb-Douglas production form would require the deriving of the cost frontier directly from the production frontier, which is not the case in this regard. The computation of AE is made on the assumption that the estimated cost frontier would be closer to the derived cost frontier.

- Across alternative functional specifications (in this case between the Cobb-Douglas and conventional translog specifications), the ranking of efficiency estimates by farms in relation to a particular distribution assumption is not affected.

These hypotheses were tested by comparing mean efficiency estimates, the distributions of efficiency levels, and pairwise Spearman's correlation coefficients of distributions for each of the Cobb-Douglas and translog specification and between the specifications for each particular distribution.

To obtain efficiency estimates, both the stochastic production and cost frontiers were re-estimated (without the inefficiency models) for the Cobb-Douglas and translog specifications, using LIMDEP (Greene, 1998). LIMDEP was preferred to FRONTIER (Coelli, 1994) for these particular estimations because it allows for estimation using all three distributions of u_i, in contrast to FRONTIER that is programmed only for the truncated-normal and half-normal distributions. LIMDEP was, however, seen as relatively disadvantageous to FRONTIER for the previous estimations because it does not incorporate the inefficiency model, which would have reverted the estimation to the criticized two-stage procedure (i.e., to firstly estimate the frontiers, then to regress the resulting inefficiency levels on the explanatory variables of the inefficiency model).

The estimation procedure entailed two steps. First, the maximum likelihood estimates of the parameters of the distributions, along with the parameters of the stochastic frontiers, β, were obtained. This step involved maximizing the loglikelihood functions with respect to the relevant parameters in relation to the half-normal, exponential and truncated normal distributions. For a sample of I producers, these loglikelihood functions are specified as follows (Kumbhakar & Lovell, 2000).

- Exponential Distribution

$$\ln L = \text{constant} - I\ln\sigma + I\left(\frac{\sigma_v^2}{2\sigma_v^2}\right) + \sum_i \ln\Phi(-A) + \sum_i \frac{\varepsilon_i}{\sigma_u} \tag{3.28}$$

where A= $-\tilde{\mu}/\sigma_v$ and $\tilde{\mu} = -\varepsilon - (\sigma_v^2/\sigma_u)$

- Half-normal Distribution

$$\ln L = \text{constant} - I\ln\sigma + \sum_i \ln\Phi\left(\frac{\varepsilon_i\lambda}{\sigma}\right) - \frac{1}{2\sigma^2}\sum_i \varepsilon_i^2 \tag{3.29}$$

Truncated-normal Distribution

$$\ln L = \text{constant} - I\ln\sigma - I\ln\Phi\left(\frac{\mu}{\sigma_u}\right) + \sum_i \ln\Phi\left(\frac{\mu}{\sigma\lambda} - \frac{\varepsilon_i\lambda}{\sigma}\right) - \frac{1}{2}\sum_i\left(\frac{\varepsilon_i + \mu)}{\sigma}\right)^2 \tag{3.30}$$

where $\sigma_u = \lambda\sigma/\sqrt{1+\lambda^2}$

In the second step, the estimates of technical (economic) efficiency were computed from the conditional distribution of μ_i given ε_i (Eqn. 2.11), from which the following means of the three distributions were used as point estimators for u_i.

- Exponential distribution: *f(u|ϵ)* is distributed as $N^+(\tilde{\mu}, \sigma_v^2)$, with mean

$$E(u_i|\epsilon_i) = \tilde{\mu}_i + \sigma_v\left[\frac{\phi(-\tilde{\mu}_i/\sigma_v)}{\Phi(\tilde{\mu}_i/\sigma_v)}\right] \tag{3.31}$$

- Half-normal distribution: *f(u|ϵ)* is distributed as $N^+(\mu_*, \sigma_v^2)$, with mean

$$E(u_i|\epsilon_i) = \mu_{*i} + \sigma_*\left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1-\Phi(-\mu_{*i}/\sigma_*)}\right]$$

$$= \sigma_*\left[\frac{\phi(\varepsilon_i\lambda/\sigma)}{1-\Phi(\varepsilon_i\lambda/\sigma)} - \left(\frac{\varepsilon_i\lambda}{\sigma}\right)\right] \tag{3.32}$$

where $\mu_* = -\varepsilon\sigma_u^2/\sigma^2$ and $\sigma_*^2 = \sigma_u^2\sigma_v^2/\sigma^2$

- Truncated-normal distribution: $f(u|\epsilon)$ is distributed as $N^{+}(\tilde{\mu}_{*}, \sigma_{*}^{2})$, with mean

$$E(u_i|\epsilon_i)=\sigma_{*}\left[\frac{\tilde{\mu}_{*}}{\sigma_{*}}+\frac{\phi(\tilde{\mu}_{*}/\sigma_{*})}{1-\Phi(-\tilde{\mu}_{*}/\sigma_{*})}\right] \tag{3.33}$$

where $\tilde{\mu}_{*}=(-\sigma_u^2\varepsilon_i+\mu\sigma_v^2)/\sigma^2$ and $\sigma_{*}^2=\sigma_u^2\sigma_v^2/\sigma^2$.

From the point estimates obtained from estimations of equations 3.31-3.33, estimates of technical (economic) efficiency for each producer were computed as $TE_i=\exp\{-\hat{u}_i\}$ (Eqn. 2.2), or $EE_i=\exp(\hat{u}_i)$ (Eqn. 2.24), where $\hat{u}=E(u_i|\epsilon_i)$ (eqns. 3.31-3.33).

3.2 Sources and Construction of Data

The data for this study consist of input and output quantities, their prices, and expenditure on miscellaneous items, for milk production from a representative panel of Alberta milk producers. These data were obtained from the Production Economics Branch of Alberta Agriculture from a survey on Alberta dairy cost of production for the years 1980 to 1996 inclusive. The survey is administered by Alberta Agriculture officials, in conjunction with the Alberta Milk Producers' Society, who choose a sample of fluid milk producers to complete and return a monthly questionnaire recording information related to the previous month's milk production and costs. This sample is designed to be representative in terms of spatial and size distribution of the dairy herds in Alberta.

The sample size is determined statistically every year. In this panel it ranges from 54 to 74 cross sectional observations per year, which sum up to 1046 observations for the whole panel data set. The data set is unbalanced in that not all producers participated for the whole period and the sample size varies from year to year. Since producers participate by their own volition, they are free to opt out of the survey at any time. Producers who opt out of the survey are replaced with participants from the same

(Northern or Southern) region owning a similar amount of fluid quota. The average period of participation in the survey is four years.

In the survey, information concerning farm identification is provided. However, within the data set itself, each observation has an ID number that is related to the year in which the survey was conducted, and not to a particular producer. As a result, it was not possible to track the relative performance and efficiency of specific farms over time.

This very extensive set of data is collected and recorded in 164 disaggregated categories for each farm in every year. The information in these categories can be broadly grouped as follows: dairy herd, capital purchases, sales, feeds, labour, other expenses, farm loans, supplies, machinery and equipment, buildings, land (including building sites and pasture), and milk quota. It is gathered through two questionnaires - one on sales, purchases and other expenditures and another on investment.[39] From the survey, data are processed and aggregated into annual values. The data set is stored in the data bank of the Production Economics branch of Alberta Agriculture. It is from these annual summaries that the quantities and value of the variables for the models, previously discussed, are computed.

3.2.1 Quantities of Output and Inputs

The categorising of the inputs drew mainly from previous studies that have used the same data, albeit for shorter periods (Richards & Jeffrey 1996, 1998; Richards 1993, 1995). On the input side, five aggregate quantities of inputs per cow per year were specified: grains and concentrates (GC), hay and forage (HF), labour (L), capital (K), and other intermediary inputs (OI).

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[39] The survey collects information on assets and debt as well as supplies. Investment in the questionnaire is interpreted in the accounting sense in that it is recorded as $ amount incurred rather than computed from shadow prices of the resources.

The quantities of grains and concentrates, and hay and forage were obtained by summing up the quantities (in tonnes) of feeds for each category. The grains and concentrates feed group included oats, barley, wheat, grain, beet pulp, dairy ration, milk replacer, supplements, molasses, salt, and minerals. The hay and forage group included alfalfa hay, alfalfa pellets, straws, green feed and silage. No distinction was made between grain and hay grown on farm and purchased feed. As Richards (1993) argues, any differential in feed quality should be reflected in relative price. As well, the value of pasture usage was ignored, being an insignificant feed input in Alberta's milking herd.[40]

The quantity of labour was calculated as follows: first the average wage rate was calculated for the three types of labour, viz., hired, operator and family labour. Then the total wage bill for the three types of labour was divided by the average wage rate to obtain the total quantity of composite labour that reflects remuneration structure for each of the three labour types. The amount of operator and family labour is defined as hours worked per year according to the respondent's recorded amount. Hired labour per farm is recorded at the actual hours used and the total wage paid, including common benefits such as room and board. However, the data on hired labour has missing values for some farms on hours worked or wage, or both. If information for both wage and hours worked is missing, the assumption was made that that particular farm did not hire labourers. However, if only one of the two is missing, it was assumed that the farm hired labourers but did not give complete information.

In dealing with this problem of unrecorded information, the average of hours worked or wage paid was first computed from fully recorded information and then used to fill in the missing entries[41]. The averages for the whole sample were then re-computed.

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[40] Richards (1993) suggests this reason for ignoring pasture. A significant number of zero entries in the data for this category supports this assumption.

Although treating labour as one aggregate variable input differs from some studies that have categorised labour into two separate variables (e.g., Richards, 1993; Richards & Jeffrey, 1996), it shares commonality with other studies (e.g., Moschini, 1988). Dividing labour into two separate groups would require one to justify the distinction on both theoretical and empirical grounds as to why one category of labour, under the same conditions, would be significantly more productive than the other. Moreover, it is impossible to split the contribution of various types of labour in the composite output, especially because in the data utilised the variables are measured with the same units.

The value of dairy capital is used as the capital input in the production frontier. Capital consists of the value of buildings, machinery, and equipment specific to the dairy enterprise, a proportion of non-dairy equipment, and land allocated for dairy activities, and livestock. Livestock is included in the capital aggregate, rather than being treated as a separate input, because its services are not exhausted in a single year. The size of the livestock herd (cows and heifers)[42] was calculated as the average of the starting and year-end numbers. The value of livestock in current terms was obtained by multiplying the calculated average number of animals for each category of livestock by the respective annual average price for each type of animal.

Using the input price deflators, derived from respective farm input price indexes (Statistics Canada, 1998/1999), the series of different categories of capital were converted to constant values and summed. As with other categories, for items categorised as capital the data were provided for every year, and were varying. This was taken to imply that the farmers adjust their stock of capital every year to the amount that is consistent with optimisation. As a consequence, it should be perceived that long run frontiers were estimated.

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[41] An alternative would have been to delete the observations. Thus the choice was made based on the need to recover data rather than lose data, so as to minimize bias in the estimates.

[42] In addition to cows and heifers, the dairy enterprise includes also bulls and young animals.

Other intermediate inputs include all other inputs and expenses not in the categories already specified. They comprise processing costs, veterinary and medicine, milk hauling, producer fees, utilities, fuels, and miscellaneous expenses. These, being given in value terms, were summed up and deflated by the farm-input price index to get the total value in constant terms, which was used as a proxy for quantity of other intermediate inputs. Lastly, the quantity of milk produced (in hectolitres) is calculated as the sum of quota milk, over-quota milk, milk fed to livestock, and other milk sales.

3.2.2 Variables for the Technical (Economic) Inefficiency Model

To explain technical inefficiency, the data utilised variables that are computed or given directly from the sample data. Those taken directly from the sample data include years of farming, breeding and veterinary expense, and the trend variable. The ratio of capital to labour and the ratio of grain and concentrate to hay and forage were computed using the respective variables discussed earlier, whereas the ratio of the number of milk cows to total number of cattle was computed from information taken directly from the sample data. These data variables are discussed, in terms of their relevance, in section 3.1.1.

3.2.3 Input Prices and Cost of Production

In Alberta, input prices vary due to geographical diversity of producers and the marked differences in local markets (Richards, 1995). The wage rate for the composite labour was obtained by averaging the individual wage rates for operator, family and hired labour. Wage is unadjusted for quality due to lack of data concerning worker training and experience. Prices of grain and concentrates and hay and forage were computed as weighted average price per tonne of feed used (calculated by dividing the total number of tonnes by the sum of the prices). For home-grown feed the computations are based on regional average prices whereas purchased feeds are valued at local market prices. For other intermediate (OI) aggregated inputs, the farm input price deflator was used as a proxy for its price.

For capital, since farmers typically own the inputs, the price is imputed, based on the notion of user cost of capital. The user cost of capital is defined as follows (Moschini, 1988):

$$r_j = R_i(i + d_j + \tau_j - \rho_j)$$

where *j* indexes the capital input,

r=the user cost (rental price),

R=the capital (replacement) price,

i=the interest rate (the opportunity cost of holding capital)

d=the physical depreciation rate

τ=the tax rate for capital, and

ρ= the expected rate of change of R (expected capital gain).

Since annual data were used, expected capital gain was not considered. The tax rate (τ) was not provided in the data set. Therefore, it was calculated by dividing total tax paid (i.e., primarily property tax) by total capital. Physical depreciation was partly given in the recorded information, consisting of depreciation for buildings, machinery, and equipment. Physical depreciation for each of these capital items was divided by corresponding capital values to obtain the depreciation rates. These rates closely approximate those used by Moschini (1988)[43].

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[43] The physical depreciation rates used in Moschini (1988) are 0.15 for machinery, 0.05 for buildings, and zero for land and livestock.

Following Ball (1985), depreciation for livestock was estimated based on the difference between the acquisition price of the animal and the salvage or slaughter price. These data, however, were not recorded in the sample. Instead, they was compiled from *Economics of Milk Production* (Alberta Agriculture, Production Economics Branch, various issues). The difference between sample averages of the acquisition price and salvage price was computed, which formed the basis for approximating its proportion to the value of livestock. To reduce the effect of stochastic market conditions, a three-year moving average was applied to the computations. These were multiplied by the percentage replacement, approximated as one third of the herd (Durr et al., 1997; Kulak et al., 1997) to get the depreciation rate.

The rate for 90-day Treasury bills was used as a proxy for the user cost of capital (Statistics Canada, 1998/1999). This was taken to approximate the risk free rate. The risk free rate was deemed to be appropriate because dairy farming is perceived as a very low risk undertaking as compared to other farm businesses. According to Agriculture and Agri-Food Canada, "real dairy farm cash receipts have grown very moderately, at a more stable, but much slower, pace than the agricultural sector as a whole" (Agriculture and Agri-food Canada, 1996, p. 28)[44].

Capital replacement value is measured by the farmer-owned capital stock. Thus the rates above are applied to this value to give the rental price. However to obtain the full measure of capital costs, these values were added to rental value and repairs to buildings and machinery (Moschini, 1988).

Other inputs (OI), were primarily recorded in monetary terms were deflated mainly by the respective price indexes of the input price index to get a measure that represents their quantity. The values of those inputs for which own price indices were not found were deflated by the general input price index.

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[44] For more than 30 years from 1960, dairy cash receipts (in 1986 constant prices) have remained at approximately $5million (Agriculture and Agri-Canada, 1996, p.28).

Total cost of production of milk per year per cow from a sample of Alberta dairy farms was computed as a sum of costs for grain and concentrates, forage and hay, labour, capital and other inputs. Total cost for each farm in every year was obtained by multiplying input quantities by the respective input prices. Appendix C and Table 3.1 provides summary statistics of the data used.

Table 3.1 *Summary Statistics for Selected Variables*

	Quantity of Milk (Hectolitres/per cow	Total Cost in $ per cow	Capital $ per cow	Herd size
Mean	69.5	2143.9	2669.3	79.89
Variance	223.1	543776.3	676749.8	1701.3
Standard Deviation	14.9	737.4	822.6	41.25
Maximum	177.6	8663.4	7397.4	281.50
Minimum	32.8	841.6	888.3	24.50
Count	1046	1046	1046	1046

Chapter 4. Estimation and Results

4.1 Estimated Models

The following sections report on results of two functional forms for both the production and cost stochastic frontiers. These include a simplified translog and a Cobb-Douglas specification. Each of these specifications was estimated twice. The first set of estimations (Estimation 1) covers the period 1980 to 1996, which is the entire sample period. The second set of estimations (Estimation 2) covers a subset of the entire period, running from 1986 to 1996. Estimation 2 includes years of farming as one additional variable that explains technical efficiency. This variable was not available for the entire sample period of 1980 - 1996. Estimation 1 has the advantage of a larger sample size, which increases the degrees of freedom, hence the efficiency of the estimation, while Estimation 2 has the advantage of the added explanatory variable, which is hypothesized to be important in explaining the dependent variable.

Estimation of the conventional translog is generally prone to problems of multicollinearity (Cornwell et al. 1990; Ahmad & Bravo-Ureta, 1996), which may affect the size, sign, and statistical significance of the coefficients. For example, the conventional translog cost frontier estimated by Richards and Jeffrey (2000) violated concavity, and thus could not be solved for optimal input levels. In the present study, first, for both production and cost frontiers the majority of the coefficients estimated for the translog formulation were not statistically significant. Second, for the production frontier, the output elasticities with respect to some of the inputs were found to be negative[45]. Third, the translog cost frontier could not be solved for input prices and input levels at the point where farmers are technically but not allocatively efficient; that is, the parameter estimates failed to converge at that point. The results of the translog are summarized in Appendix 1.

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[45] Negative elasticity estimates imply, contrary to theoretical expectations, that the increased utilisation of the inputs in question results in reduced level of output, that is, the "average" farmer would fare

A simplified translog frontier function was therefore specified as presented below and estimated. This specification was used by Ahmad and Bravo-Ureta (1996) in place of a conventional translog formulation that did not give satisfactory results; and they obtained results that were closer to the Cobb-Douglas formulation with regard to the statistical significance of the estimated coefficients. The simplified translog formulation is essentially the restricted form of the conventional translog whereby the interaction between inputs (in the case of production frontier) or input prices (in the case of cost frontier) are eliminated but an interaction of time with inputs or input prices is included. The assumption for excluding interactions among inputs or input prices while introducing interactions of time with inputs or input prices is that inputs or input prices are separable from each other but not from time (Fan, 1991).

Following Ahmad and Bravo-Ureta (1996), the simplified translog for the production frontier is specified as:

$$\ln Y_{it} = \beta_0 + \sum_k \beta_k \ln X_{kit} + \sum_k \beta_{tk} \ln X_{it} T + \beta_t T + \beta_{tt} T^2 + v_{it} - u_{it} \qquad (4.1)$$

where β's are parameters to be estimated, X_k are inputs, T is a smooth time trend accounting for technological change and v_{it} and u_{it} are the error terms of the stochastic production frontier.

Both the simplified translog and Cobb-Douglas formulations include a smaller number of terms in the specifications, which is likely to reduce collinearity between

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better in terms of output produced if those inputs were reduced, implying that the farmer is operating in stage three of the production process.

variables, although these forms are less flexible specifications than the conventional translog.

4.2 Production Frontier Results and Discussion

Coefficient estimates of the simplified translog production frontier have no specific economic interpretation, unlike those of the Cobb-Douglas production frontier[46]. These coefficients were used to derive the output elasticities of inputs and Returns to Scale (RTS), which are presented in Section 4.2.1 along with those from the Cobb-Douglas production frontier. On the basis of the analysis of elasticities of inputs and Returns to Scale (RTS), specific insights into the production structure of the Alberta dairy sector are provided in the following section.

In addition to elasticities and RTS, estimated technical efficiency (TE) measures are reported. The mean, median and overall distribution (in percentage of farms) of TE were used to gauge the extent of TE (Section 4.2.2). The results of the Likelihood ratio (LR) test for the adequacy of functional forms are discussed in Section 4.2.3. Results of the TE model, estimated simultaneously with the production frontiers, are discussed in Section 4.2.4.

Table 4.1 and 4.2 report results from estimations of Cobb-Douglas and simplified translog frontiers. The coefficient estimates for the Cobb-Douglas production frontier for both Estimation 1 and Estimation 2 were all positive, as expected by theory, and were statistically significant at 5 percent (Table 4.1). Also, most of the coefficients in the simplified translog formulation were positive and statistically significant (Table 4.2).[47]

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[46] The following discussion is based primarily on results obtained from the estimation of the Cobb-Douglas production and simplified translog frontiers for reasons given above

[47] Note in Appendix 1a that the coefficients in the conventional translog formulation are negative and statistically insignificant at 5 percent for both Estimation 1 and Estimation 2.

Table 4.1 *Coefficient Estimates for Parameters the Cobb-Douglas Production Frontiers*

	ESTIMATION 1 1980-1996		ESTIMATION 2 1986-1996	
VARIABLE$^+$	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	3.285*	31.075	3.541*	31.273
Grains and concentrates(GC)	0.075*	4.633	0.077*	3.294
Hay and forage (HF)	0.025*	2.383	0.034*	2.993
Labor (L)	0.029*	3.886	0.020*	2.913
Capital (K)	0.039*	2.344	0.021	1.112
Other inputs (OI)	0.093*	4.581	0.096*	4.305
Year (YR)	0.018*	12.202	0.016*	7.210
σ	0.040*	7.642	0.085*	2.650
$\gamma\ (=\sigma^2_u/\sigma^2_u+\sigma^2_v)$	0.606*	9.224	0.933*	41.411

*Coefficient is significant at 5% level.

Table 4.2 *Coefficient Estimates for Parameters of the Simplified Translog Production Frontiers*

	ESTIMATION 1 1980-1996		ESTIMATION 2 1986-1996	
VARIABLE	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	2.896*	12.787	3.446*	16.295
Grains and concentrates (GC)	0.112*	4.063	0.080*	1.809
Hay and forage (HF)	0.023	1.067	0.040	1.638
Labor (L)	0.088*	4.810	0.032	1.699
Capital (K)	0.130*	3.768	0.100*	1.997
Other inputs (OI)	0.061	1.500	0.064	1.274
Year (YR)	0.053*	2.660	0.018	0.595
GC.YR	-0.004	-1.170	-0.001	-0.081
HF.YR	0.000	-0.212	0.000	-0.058
L.YR	-0.005*	-3.262	0.000	-0.164
K.YR	-0.009*	-2.744	-0.011	-1.608
OI.YR	0.003	0.682	0.003	0.453
YR.YR	0.000	0.718	0.001	1.927
σ	0.046*	4.576	0.081*	2.693
$\gamma\ (=\sigma^2_u/\sigma^2_u+\sigma^2_v)$	0.693*	9.143	0.931*	39.046

*Coefficient is significant at 5% level.

The time variable is largely positive and significant in both models, indicating the occurrence of technical change over the sample periods. In the estimated production frontiers, the time variable, YR, was used to capture disembodied, Hicks-neutral technical change, modelled as a shift in the production frontier. Both for the Cobb-Douglas and simplified translog specifications in Estimations 1 and Cobb-Douglas specification in Estimation 2, the coefficients of the partial differential of the logarithm of Q with respect to YR (which captures the rate of growth of output per unit of "aggregate input") is positive and statistically significant, which indicates that there has been significant neutral technical change in the production of milk by Alberta farmers during the period under study.

4.2.1 Elasticity of Output with Respect to Inputs and Returns to Scale

The elasticity of frontier output with respect to the k-th input for the conventional translog formulation is given as

$$\frac{\partial \ln(\overline{Q})}{\partial \ln(X_k)} = \beta_k + \beta_{kk} X_{kit} + \sum_{j \neq k} \beta_{kj} X_{jit}, \qquad (4.2)$$

where $\overline{Q}$ is the mean of milk output for the sample, X_k= the kth input and X_j= other inputs used apart from the kth input. For the simplified translog, the output elasticity is given by:

$$\frac{\partial \ln(\overline{Q})}{\partial \ln(X_k)} = \beta_k + \beta_{kk} YR_t \qquad (4.3)$$

where YR is a time trend variable, and the rest of the variables are as defined above[48]. The estimated values from eqns. 4.2 and 4.3 refer to the elasticity of best practice production with respect to the inputs involved (i.e., neutral inefficiency model).

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[48] For both the conventional translog and simplified translog formulations, the elasticities were evaluated at the mean values of the variables.

However, if the inefficiency model is specified such that it contains interaction terms between the explanatory variables of the inefficiency model and the frontier (i.e., the non-neutral inefficiency model) then the elasticity of mean output with respect to the input involved includes a component referred to as the *elasticity of the technical efficiency* with respect to the k-th input variable[49]. This component is zero for the neutral inefficiency model (Huang & Liu, 1994; Battese & Broca, 1996)[50].

The inefficiency model for this study includes the ratio of capital to labour as one of the variables explaining the inefficiency. Thus, by estimating this model simultaneously with the stochastic production frontier, both the elasticities of output for best practice (i.e., for the neutral inefficiency model) with respect to labour and capital are affected by the capital to labour ratio variable. Although the computed output elasticities relate to the neutral model, the elasticity of technical efficiency with respect to labour in this study is negative and vice versa for capital; hence the total elasticity of output with respect to labour is greater than that for best practice, and vice versa for capital.

The elasticities reported below are from the simplified translog (evaluated at the mean value for YR) and the Cobb-Douglas formulations (Table 4.3). The elasticities for the Cobb-Douglas specification are the estimated coefficients of the log-linearized production frontiers. The output elasticities from the conventional translog are presented in Appendix 1b and are not included in the discussion for reasons noted earlier.

For the Cobb-Douglas and simplified translog frontiers output elasticities in both Estimations 1 and 2 are all positive. This implies that using more of any of the inputs would lead to increased output, as theory postulates for rational producers. In discussing these results further, three sets of comparison can be made. The first is the comparison of the relative importance of each input in a given estimation; the second is the comparison between Estimation 1 (which includes years of farming in the TE model) and Estimation 2 (which does not include years of farming); and the third is the comparison of elasticity measures between the Cobb-Douglas and simplified translog formulations.

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[49] The elasticity formula for the non-neutral inefficiency model is given in Battese and Broca (1996).

[50] It has been argued that because of the correlations between the explanatory variables of the frontier and the inefficiency effects, the maximum likelihood estimators of the parameters of the Huang and Liu (1994) model would not be consistent (Battese & Broca, 1996).

The elasticities for the Cobb-Douglas specification are the estimated coefficients of the log-linearized production frontiers. The elasticities reported below are from the simplified translog (evaluated at the mean value for YR) and the Cobb-Douglas formulations (Table 4.3). The output elasticities from the conventional translog are presented in Appendix B and are not included in the discussion for reasons noted earlier.

For the Cobb-Douglas and simplified translog frontiers output elasticities in both Estimations 1 and 2 are all positive. This implies that using more of any of the inputs would lead to increased output, as theory postulates for rational producers. In discussing

these results further, three sets of comparison can be made. The first is the comparison of the relative importance of each input in a given estimation; the second is the comparison between Estimation 1 (which includes years of farming in the TE model) and Estimation 2 (which does not include years of farming); and the third is the comparison of elasticity measures between the Cobb-Douglas and simplified translog formulations.

All of the output elasticities range between 0.02 and 0.1. The highest output elasticity is obtained with respect to other inputs (OI) followed by the elasticity with respect to grains and concentrates (GC). This is consistent across estimations and specifications. Overall, the smallest output elasticity is with respect to the labour input.

Table 4.3 *Elasticity of Output with Respect to Inputs for Production Frontiers*

	COBB-DOUGLAS		SIMPLIFIED TRANSLOG [a]	
INPUT	ESTIMATION 1 (1980-1996)	ESTIMATION 2 (1986-1996)	ESTIMATION 1 (1980-1996)	ESTIMATION 2 (1986-1996)
Grains and Concentrates	0.08	0.08	0.08	0.08
Hay and Forage	0.03	0.03	0.02	0.04
Labour	0.03	0.02	0.04	0.03
Capital	0.04	0.02	0.05	0.03
Other Inputs	0.09	0.10	0.08	0.08
Returns to scale (RTS)	0.27	0.25	0.27	0.26

[a] The elasticities for the simplified translog are evaluated at the mean value for YR.

Higher output response with respect to OI and GC may indicate that purchased material inputs are utilized more productively than labour. The relatively higher values for OI and GC may simply be an indication that, given current production practices, these inputs are more productive "at the margin".

Across estimations, higher output elasticities are observed with respect to capital (K) for the Cobb-Douglas frontier, and capital (K) and Hay and Forage (HF) for the simplified translog frontier. The effects of including the experience variable and the variation in the sample size on output elasticities across estimations cannot be explained systematically, since the effects are not similar across inputs. However, lower output elasticity with respect to labour in Estimation 2 (which includes the experience variable) may indicate that some of the effects of labour are shared by the experience variable, which is embodied in labour. However, while there are numerical differences in the elasticities, they are not significant in terms of magnitude, which is also true of the comparison across functional specifications.

Comparing output elasticities across functional specifications, no systematic pattern is observed for all inputs. The elasticities of output with respect to grains and concentrates (GC) are the same in both specifications. Elasticities with respect to other inputs (OI) are higher in the Cobb-Douglas specification than in the simplified translog frontier, whereas those with respect to labour and capital are higher in the simplified translog frontier than in the Cobb-Douglas frontier.

Since the output elasticities are all positive and closer to zero than to one, it may be concluded that Alberta dairy farmers are operating within the rational economic zone; that is, in Stage II of the production process where average physical product (APP) is falling. However, Estimation 2 for the conventional translog formulation resulted in a RTS value that is greater than 1, implying increasing returns to scale (Appendix 1A).

The returns to scale (RTS) are the sum of the individual output elasticities for a given estimation. From Table 4.3, it can be observed that RTS are comparable between the Cobb-Douglas and simplified translog formulations. Theoretically, given the assumption of zero profit in the long run, the functions should exhibit constant returns to scale. However, RTS for both the Cobb-Douglas and simplified translog frontier are lower than unity, implying decreasing returns to scale (DRS). Since the elasticity estimates that resulted in DRS seem to be consistent with theoretical expectations, the results in general tend to indicate that Alberta farmers are operating in an economically rational scale[51].

4.2.2 Technical Efficiency Effects

To investigate if there exists significant technical inefficiency, the maximum likelihood estimates of the γ-parameter were used in a LR test. The γ-parameter is the ratio of the variance for the inefficiency error term (σ^2) to a sum of the variances for the total error term, $\sigma_s^2=\sigma^2+\sigma_v^2$ (Eqn. 3.5). Specifically, the test was cast as H_0: $\gamma=0$; that is, there are no significant TE effects in the production of milk by Alberta farmers.

Technical inefficiency of farms is said to be negligible the closer the γ-parameter is to zero. In the absence of technical inefficiency, all deviations are random, and the standard average function (e.g., fitted using OLS) may be used to estimate the frontier. On the other hand, as γ approaches one, the model tends to be more deterministic, but whether the deterministic frontier is appropriate depends on whether or not γ is significantly different from one.

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[51] The relation between returns to scale and profit is expressed as $\Pi=PY(1-RTS)$ where Π is profit, P is output price, Y is output, and RTS is returns to scale – a function of output. This relation implies that profit maximization is consistent with DRS. Kumbhakar argues that if efficiency and RTS vary across farms, those with lower RTS and relatively more efficiency will be more profitable (Kumbhakar, 1991).

The LR test for this hypothesis was conducted using the log-likelihood function values of the estimated stochastic frontiers and the values of the corresponding OLS production functions. More formally, the test is formulated as:

$$LR = -2(LLF_R - LLF_U) \qquad 4.4$$

where LLF_U and LLF_R are the log likelihood function values of the unrestricted (i.e., stochastic frontier) and the restricted (i.e., OLS) function, respectively.

From this test, the γ-parameter estimates for Estimation 1 and Estimation 2 of both production specifications were determined to be significantly different from zero (Table 4.4). This implies that the technical inefficiency effects are significant; that is, Alberta dairy farms are not 100 percent technically efficient and production functions estimated by OLS do not provide an adequate representation of the data[52].

For Estimation 1, the γ estimates are not close to one, which may indicate that the estimated stochastic frontier is different from the deterministic frontier. The γ estimates for Estimation 2 are very close to one, which may suggest that the stochastic frontiers for Estimation 2 are not different from their deterministic counterparts. Since the distinction between Estimation 1 and Estimation 2 is in the sample size, the results suggest that the years that were not included in Estimation 2 have contributed to the stochastic nature of Estimation 1.

4.2.3 Choice of Functional Form

In order to examine whether a particular functional form adequately represents the data used, two tests were undertaken. First, the Cobb-Douglas function, which can be derived as a special case of the conventional translog or the simplified translog, was tested to determine if it adequately represents the data. Second, the simplified translog was tested against the conventional translog to determine if it adequately represents the data.

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[52] For the Stochastic Frontier model to equal the average response function estimated by OLS regression, the scale parameter γ and all of the Technical Efficiency model (δ) parameters except for the constant should equal zero.

The results of these tests are also summarised in Table 4.4 (Null Hypotheses b). The hypothesis for adequacy of the Cobb-Douglas form was rejected in all cases but for Estimation 2 vis-à-vis the simplified translog. This implies that the CD form could not be chosen over the translog specification for both Estimation 1 and Estimation 2; neither could it be chosen over the simplified translog for Estimation 1. However, because the Cobb-Douglas formulation is the commonly used functional form, the results of the formulation are reported to afford comparison with other studies. In testing whether the simplified translog adequately represents the data, given the translog specification, the LR test resulted in the hypothesis being rejected for both Estimation 1 and Estimation 2, implying that the simplified translog also could not be chosen over the conventional translog specification.

Therefore, while it is inconclusive as to the choice between simplified translog and Cobb-Douglas formulations, the translog was found to be the best choice. However, the choice of the conventional translog needs to be qualified. As discussed earlier, most parameter estimates of the conventional translog frontiers were found to be insignificant, whereas some have incorrect signs relative to a priori expectations. Such results for the parameter estimates may be caused by problems in the data. However, the examination of whether the variables included in the production frontiers could be linearly related using auxiliary regressions showed that they were not.

Table 4.4. *Likelihood Ratio (LR) Tests of Hypotheses for Parameters of the Cobb-Douglas and Simplified Translog Stochastic Production Frontiers*

		COBB-DOUGLAS		SIMPLIFIED TRANSLOG	
HYPOTHESIS		Estimation 1 (1980-96)	Estimation 2 (1986-96)	Estimation 1 (1980-96)	Estimation 2 (1986-96)
a) $H_0:\gamma=0$	LLF_U[a]	**523.61**	**406.73**	**534.41**	**411.33**
Estimated frontier not different from OLS (average response) function.	LLF_R[a]	485.42	337.77	494.59	344.60
	λ_{LL}[b]	76.37	137.92	79.65	133.46
	Critical Value (5% Level)	14.85*	13.4*	14.85*	13.4*
	Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0
b)(i)$H_0:\beta_{ij}=0$ $i\leq j=1,2,\ldots,6$	LLF_U	**566.14**	**431.87**	**566.14**	**431.87**
(ii) $H_0:\beta_{ij}=0$ $i\leq j=1,2,\ldots,5$	LLF_R	523.61	406.73	534.41	411.33
Interaction terms of the conventional translog are all equal to zero, implying (i) CD and (ii) Simplified translog**	LR	85.06	50.28	63.45	41.08
	Critical Value (5% Level)	32.67*	22.36*	32.67*	22.36*
	Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0
c) (i) $H_0:\delta_1=\delta_2=\ldots=\delta_5=0$ (Estimation 1)	LLF_U	**523.61**	**406.73**	**534.41**	**411.33**
(ii) $H_0:\delta_1=\delta_2=\ldots=\delta_6=0$ (Estimation 2)	LLF_R	514.84	384.92	523.01	389.25
	LR	17.54	43.62	22.80	44.16
All parameters on the variables explaining technical efficiency are simultaneously equal to zero (i.e., no TE effects)	Critical Value (5% Level)	11.07*	12.59*	11.07*	12.59*
	Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0

* Critical Values are obtained from Kodde and Palm (1986). These values entail a mixed χ^2 distribution. Because $\gamma=0$ lies on the boundary of the parameter space for γ, the LR statistic for testing if H_0: $\gamma=0$ is true has an asymptotic distribution that is a mixture of Chi-square distributions (Coelli et al. 1998).

**The hypothesis of whether the interaction terms with time of the simplified translog are all equal to zero (i.e., $H_0:\beta_{it}=0$, $i=1,2,\ldots,6$) was rejected in Estimation 1 (1980-96) and was not rejected in Estimation 2 (1986-96).

[a] LLF_U and LLF_R are the log likelihood function values of the unrestricted and the restricted function, respectively.

[b] LR is the computed Likelihood Ratio value

4.2.4 Extent of Technical Inefficiency

This sub-section assesses the extent of technical efficiency by considering the mean, the median and the distribution (in percentage of farms) of TE among the sample of Alberta dairy farms. These measures were computed from results on TE estimates for individual farms. The mean TE shows the extent of technical efficiency of farms on average. Table 4.5 shows that the mean TE ranges between 0.87 and 0.92 for both Estimation 1 and Estimation 2. These results imply that Alberta farmers, by utilising the same amount of inputs more efficiently, could improve the average output of milk by up to 9 percent (Estimation 1) or by up to 13 percent (Estimation 2).

Since the mean TE's are not independent, a Wilcoxon test, which tests mean ranks rather than levels, was used to check for equality of mean TE's between Estimation 1 and Estimation 2 and between specifications. The null hypothesis that the mean ranks of the two tested sets do not substantially differ was rejected for all sets, except for the pair of mean TE in Estimation 2 between Cobb-Douglas and simplified translog specifications.

The results of the Wilcoxon test show consistency with the distributions of TE estimates in Figure 4.1. Across the two specifications, the charts indicate the difference between the distributions in Estimation 1 and Estimation 2. Whereas at least 68 percent of the farms are operating at 90 percent or more level of technical efficiency in Estimation 1, a slightly lower performance is observed in that at least 50 percent of the farms are operating at 90 percent or more in Estimation 2. Similarly, a higher percentage of farms with TE levels of 80 percent or more is observed in Estimation 1 than Estimation 2. These estimations are based on different sample sizes; hence, the years excluded in a shorter sample may have entailed some factors that may have affected the estimates.

Moreover, the estimations for the simplified translog (Eqn. 4.1), which show similar pattern in the distribution of TE, were not rejected by the null hypothesis in the Wilcoxon test, which contrasts with estimations of the Cobb-Douglas, whose null hypothesis in a similar test was rejected. Furthermore, Table 4.5 shows the extent of TE as gauged on the median of TE. In both Estimation 1 and Estimation 2 across both specifications, the median TE is slightly larger than the mean, implying that more than half of the farms are technically more efficient than the "average" farm. These results in general point to the homogeneity of performance among Alberta dairy farmers, which is further evidenced by a very small variance in the mean of TE's that is in the order of less than 0.005 in Estimation 1 and 0.01 in Estimation 2, respectively (Table 4.5).

4.2.5 Technical Inefficiency and Explanatory Variables

To understand potential sources of technical inefficiency for Alberta dairy farms, both the overall significance of the model for explaining TE and the significance of coefficients for the explanatory variables of the model were examined (Eqn. 3.5). The overall significance of the TE model involved tests of the three stochastic frontier production function (Cobb-Douglas, simplified translog and conventional translog) specifications for both Estimation 1 and Estimation 2. The null hypothesis was specified as follows: $H_0: \delta_i = 0, \forall_i$, ***i=1,2,3,4,5 for Estimation 1 or i=1,2,3,4,5,6 for Estimation 2*** [53]. In other words, the coefficients for the variables explaining technical inefficiency in the TE model are simultaneously zero.

The above hypothesis was tested using a Likelihood Ratio (LR) test, in which the restricted TE model has only the constant term. The restricted form implies that the combined effect of the explanatory variables on the TE of farms is insignificant. The results for the Cobb-Douglas and simplified translog specifications are shown in Table 4.4, Null Hypothesis (c). In both Estimation 1 and Estimation 2, the null hypothesis was rejected for both specifications, indicating that the TE model is statistically significant in explaining the causes of technical inefficiency of Alberta dairy farms[54].

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[53] The variables in Estimation 1 include the ratio of Grains and Concentrates to Hay and Forage (GH), the ratio of Capital to Labour (KL), Breeding and Veterinary services (BE), Time variable (YR), and Herd size (HS). Estimation 2 includes, Years of Farming (YF) in addition to the variables included in Estimation 1.

Following the existence of TE in the production of milk by Alberta dairy farmers, the discussion below elaborates on the influence of factors in the TE model in explaining the technical efficiency of Alberta milk producers. A summary of the results is provided in Table 4.6.

4.2.5.1 Capital to Labour Ratio

The coefficient on the capital-to-labour ratio variable (KL) is positive but statistically insignificant in both Estimation 1 and Estimation 2. A positive sign on this coefficient indicates that farms using less capital relative to labour tend to be more technically efficient. However, since the coefficient is statistically insignificant, the results are inconclusive as to whether greater relative capital intensity tends to enhance technical efficiency.

4.2.5.2 Ratio of Grain and Concentrates to Hay and Forage

The coefficient on the ratio of grain and concentrate to hay and forage (GH) is negative and statistically significant for both specifications and both estimations. It was conjectured in Section 3.1.1 that the effect of grains and concentrates could be either positive or negative on milk production. The above results indicate that Alberta dairy farmers can enhance technical efficiency by increasing this ratio.

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[54] The hypothesis was also rejected in the conventional translog specification.

Table 4.5 *Technical Efficiency (TE) Measures from Cobb-Douglas and Simplified Translog Production Frontiers*

	COBB-DOUGLAS		SIMPLIFIED TRANSLOG	
	Estimation 1 (1980-96)	Estimation 2 (1986-96)	Estimation 1 (1980-96)	Estimation 2 (1986-96)
Mean TE	0.91	0.87	0.92	0.88
Median	0.93	0.91	0.93	0.91
Max. Value	0.98	0.98	0.99	0.98
Min. Value	0.49	0.47	0.59	0.52
Variance	0.004	0.01	0.003	0.01

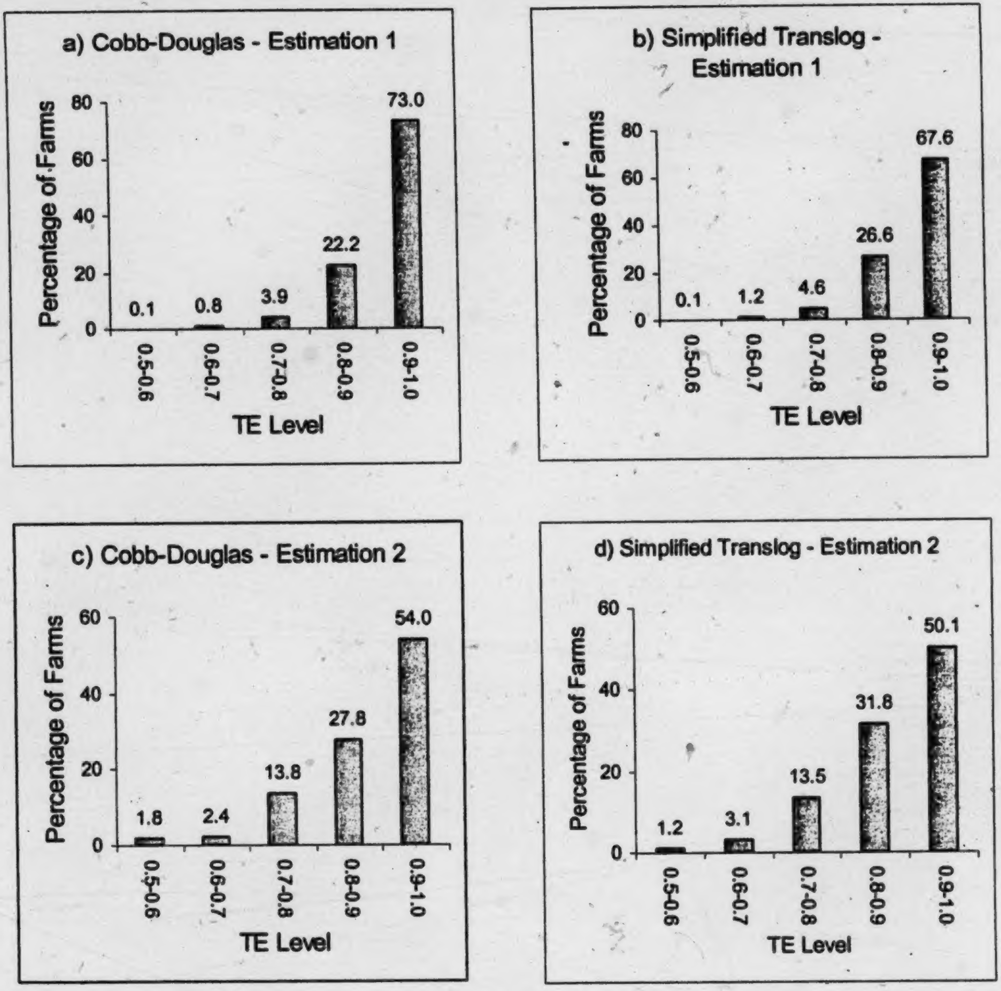

Figure 4.1 Distributions of Technical Efficiency (TE) in Percentage Terms, by Functional Form and Estimation

These results, however, are predicated on the assumption that there is no variability in the sample as regards to herd health. The interaction term between this variable and herd health would have given more insights into the effect of herd health on technical efficiency, but it was not included due to lack of data. The significance of GH variable ties in with the earlier result concerning the production elasticity for grains and concentrates (GC).

4.2.5.3 Breeding and Veterinary Services

The coefficients for the breeding and veterinary services variable (BE) in all estimations are significant and negative, implying that farms that have higher breeding and veterinary services per cow tend to be more technically efficient. Higher veterinary services lead to healthier cows whose output of milk is closer to the frontier than for those utilising lesser amounts of these services. This effect, however, may also be due to investment in improved genetics; that is, the use of higher quality semen resulting in a greater breeding cost (Richards & Jeffrey, 2000).

4.2.5.4 Time Variable

The coefficient on the time variable (YR) is insignificant in all estimations. This implies that technical efficiency for Alberta farms is likely to have remained more-or-less the same over the period examined in the study. However, the impact of time is captured in the production function by technological change, as the time variable in the production frontier models is positive and significant.

Table 4.6 *Coefficient Estimates for Models Explaining Technical Efficiency*

	COBB-DOUGLAS				SIMPLIFIED TRANSLOG			
	Estimation 1 (1980-96)		Estimation 2 (1986-96)		Estimation 1 (1980-96)		Estimation 2 (1986-96)	
VARIABLE[a]	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO
CONST.	0.310	5.757	0.134	0.773	0.277	3.532	0.137*	0.879
GH	-0.143*	-2.973	-0.241*	-2.543	-0.083*	-2.070	-0.230*	-2.752
KL	0.013	1.210	0.035	1.166	0.025	1.993	0.038	1.397
BE	-0.011*	-4.012	-0.007*	-2.299	-0.012*	-3.094	-0.007*	-2.278
YR	0.005	1.032	0.000	-0.020	0.001	0.072	-0.001	-0.099
YF	-	-	0.009*	2.490	-	-	0.009*	2.666
HS	-0.001	-1.756	-0.001	-1.517	-0.001	-1.659	-0.001	-1.680

*Coefficient is significant at 5% level.

[a] The abbreviations for the variables are as follows:
GH – the ratio of Grains and Concentrates to Hay and Forage; KL – the ratio of Capital to Labour; BE – Breeding and Veterinary services; YR – Time trend; YF – Years of Farming; HS – Herd size.

4.2.5.5 Herd Size

The coefficient on herd size is negative, implying a positive relationship between herd size and technical efficiency. However, this variable is insignificant at the 5% level in both the Cobb-Douglas and simplified translog specifications. The insignificant relationship between herd size and technical efficiency could be related to how herd size is incorporated in the models, which represents merely the quantity but not the quality of cows. With higher quality, it can be expected that same number of cows can produce more output or same amount of output can be produced by smaller number of cows. Consequently, a more definitive relationship between herd size and technical efficiency would be captured only if a quality-of-cow variable were explicitly included in the models. Alternatively, it may simply be that efficiency is not affected by herd size, or at least not given the range of herd sizes in the sample (e.g., may be if much larger herd sizes were present, we might see an effect).

4.2.5.6 Years of Farming:

The coefficient on the years of farming variable is significant and positive in all three specifications, indicating that farmers with fewer years of farming tend to be more technically efficient. These results are contrary to a priori expectations. Ordinarily, one would expect that more years in farming business would lead farmers involved to learn by experience and improve on their production.

However the above results may be rationalised if years of farming is linked to a generation gap. Using the age of farmers as one of the variables to explain technical efficiency, Seyoum et al. (1998) found a negative and significant effect, implying that younger farmers were more technically efficient. Since farmers with more years of farming are likely to be older, the results of this study tend to point to the dynamism of younger farmers in that they are likely to be more energetic, more educated, and more enterprising than older farmers. As a result, they may have a greater propensity to seek out and adopt new technologies and new management practices, for example.

4.3 Stochastic Cost Frontier Model Results

This section analyses results from the stochastic cost frontiers, in the same way as those from stochastic production frontier estimation in the previous section. In Section 4.3.1, the elasticity of cost with respect to prices of inputs are reported and discussed. Section 4.3.2 discusses the results from testing for the presence or absence of economic inefficiency in the production of milk by Alberta dairy farmers are discussed. Section 4.3.3 analyses the extent of economic efficiency as gauged by mean EE, median EE and the distribution (in percentage of farms) of EE. Section 4.3.4 is a presentation and discussion of the Likelihood ratio (LR) test results for the adequacy of functional forms. The results of the EE model, estimated simultaneously with the frontiers, are discussed in Section 4.3.5. Section 4.3.6 presents and discusses measures of allocative efficiency (AE).

As in the case of production frontiers, the discussed results are primarily for the Cobb-Douglas and simplified translog frontiers. Most of the coefficient estimates for the parameters of these cost frontiers are positive and statistically significant at 5 percent level (Tables 4.7 & 4.8). The results from the conventional translog cost frontiers, whose coefficients were mainly negative and statistically insignificant at the 5 percent level, are summarized in Appendix 1B.

4.3.1 Elasticity of Cost with Respect to Input Prices

The own price elasticities for inputs are high, ranging from –0.55 – -0.60 for Other inputs (OI) to -0.89 – -0.84 for capital (K) (Table 4.7). Except for the elasticity of hay and forage that is far too high, the results in the current study may be compared to the elasticities reported by Richards and Jeffrey (2000). However, the elasticity of labour demand from this study (-0.66) is slightly lower (i.e., slightly inelastic) than the value reported in Richards and Jeffrey (2000) (-0.815).

The derived returns to scale from the inverse of the differential of the cost frontier with respect to output shows that the cost function exhibits increasing returns to size. This is inconsistent with the results obtained from the production function in Section 4.2.1. However, consistency of these results is ordinarily assured for the cost function derived from the production frontier using duality. While it is unclear as to what caused the inconsistency, it may be conjectured that this may be related to the estimating of the two frontiers separately. Hence, the estimates are likely to have been affected by estimation of and data problems, for example, the Greene problem or the estimation of a single equation rather than a system of equations for cost frontiers.

4.3.2 Economic Efficiency Effects

The maximum likelihood estimates of the γ-parameter were used, as in the production frontiers, to assess the presence or absence of EE effects; that is, a combination of technical and allocative inefficiency. The presence of economic inefficiency is indicated by a positive statistically significant value of the γ-parameter. A deterministic frontier is appropriate if values of γ are not, in a statistical sense, significantly different from 1. Economic inefficiency was expected since the results of a similar hypothesis in the production frontiers above had already indicated the presence of technical inefficiency, which is a component of economic inefficiency.

Table 4.7 *Elasticity of Cost with Respect to Input Prices for Alberta Dairy Farmers*

	COBB-DOUGLAS		SIMPLIFIED TRANSLOG[a]	
PRICE OF:	ESTIMATION 1 (1980-1996)	ESTIMATION 2 (1986-1996)	ESTIMATION 1 (1980-1996)	ESTIMATION 2 (1986-1996)
Grains and Concentrates	-0.83	-0.88	-0.84	-0.88
Hay and Forage	-1.014	-0.99	-1.014	-0.98
Labour	-0.66	-0.65	-0.68	-0.65
Capital	-0.89	-0.89	-0.87	-0.84
Other Inputs	-0.60	-0.55	-0.58	-0.61

[a] The elasticities for the simplified translog are evaluated at the mean value for YR.

In all of the estimations, the γ-parameter estimates were found to be very large and statistically significant at a 5% level (Tables 4.8 & 4.9), an indication that economic inefficiency effects are very significant in the analysis. These t-test results were further confirmed with the LR-test for the presence of EE effects in which the null hypothesis, H_o: γ=0 was tested; that is, there are no significant EE effects in the cost of production of milk output by Alberta dairy farmers. In this LR test (see Eqn. 4.4), the unrestricted log likelihood functions (LLF_U) were obtained from the full stochastic cost frontiers and the restricted ones (LLF_R) from the corresponding OLS cost functions. The interpretation is similar as in the case of production frontiers: with no EE effects, the variance of *u* is zero, and the deviation in cost of production is attributed wholly to randomness. Hence, the estimation of cost frontiers by OLS would be appropriate.

The hypothesis was rejected in all estimations (Table 4.10 Hypothesis a), which implies that the Alberta dairy farms are not fully economically efficient. This may be primarily due to technical inefficiency, calculated earlier, or may also be caused by allocative inefficiency, which is examined below (section 4.3.6). The rejection of the hypothesis implies also that the cost functions estimated by OLS are not adequate representations of the data.

The t-test for the null hypothesis that γ estimates are equal to one indicates that in all estimations, the estimates are significantly different than 1 (t= -5.13 and –4.17 in Cobb-Douglas and –5.34 and -4.81 in simplified translog frontiers). This indicates that the estimated stochastic cost frontiers are significantly different from their respective deterministic kernels. The implication is that, during the time period considered, random factors were significant in the discrepancy between actual and optimal costs of production for Alberta dairy farmers.

4.3.3 Choice of Functional Form

An LR test was conducted to examine the appropriateness of the Cobb-Douglas and simplified translog cost frontiers relative to the conventional translog specification[55]. coefficient estimates of the frontiers[56]. Consequently, the results of the conventional translog cost frontier are not reported or discussed in this section.

4.3.4 Extent of Economic Inefficiency

The extent of economic efficiency of farms is indicated by the mean and median EE (Table 4.11) and the distribution of EE (Figure 4.2), computed from the results of EE estimates on individual farms. The estimate of average economic efficiency of farms (mean EE) is approximately 83-84 percent. This implies that the average farm could reduce costs by approximately 16 percent if it were to improve its use of inputs so as to get maximum output as well as to allocate them to incur the least cost.

The Spearman rank correlations of EE levels between Estimation 1 and Estimation 2 of the Cobb-Douglas and simplified frontiers and across the two specifications are very high. The Wilcoxon test for the equality of the EE distributions indicates, however, that the paired distributions are not exactly the same (p-value 0.001). The median of EE is above the mean EE in all estimations (i.e., approximately 86 percent versus 84 percent, respectively). This indicates that more than half of Alberta dairy farms are operating at economic efficiency levels that are higher than the "average".

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[55] The hypothesis whether the interaction terms with time of the simplified translog are all equal to zero (i.e., H_0:β_{it}=0, i=1,2,...,6) was also tested. It was rejected in Estimation 1 (1980-96) and was not rejected in Estimation 2 (1986-96).

[56] In testing the equivalence of the Cobb-Douglas formulation to the conventional translog, all second order coefficients of the conventional translog were restricted to zero and the log-likelihood function value (LLF_R) was compared to the log-likelihood function value of the unrestricted form (LLF_u). For the equivalence of the simplified translog to the conventional translog, the test involved the restriction of the second order coefficients to zero except those of the time trend. The resulting log-likelihood function values from the restricted and unrestricted form were compared in a LR test. The LR test rejects the equivalence of the Cobb-Douglas and the simplified translog cost frontiers to the conventional translog cost frontiers. The test results are indicated in Table 4.9.

The results, as in the case of the production frontiers, favoured the conventional translog form. However, this functional form did not result in satisfactory results with regard to the farm; that is, in terms of both physically utilising the inputs and in allocating them according to their relative prices. In terms of the percentage distribution of EE levels (Figure 4.2), 70 percent of the farms are operating at 80 percent level of economic efficiency or higher, and 90 percent of the farms at 70 percent level or higher. Only about 5 percent or less are below the 60 percent efficiency level. In addition to efficiency measures being predominantly in the 70-99 percent range, the distribution of EE levels is characterised by low variance (i.e., less than 1.5%), which is an indication of a high degree of homogeneity of performance among Alberta herds.

4.3.5 Economic Efficiency and Explanatory Variables

Given that the LR test has indicated the presence of economic inefficiency, the discussion in this section addresses the economic efficiency model. In particular, is the Economic Efficiency (EE) model (Eqn. 3.13) significant and, if so, what factors are individually significant in explaining the inefficiency? The statistical significance of the EE model is established using a joint F-test for the coefficients of the model with the effect that if the explanatory variables are jointly statistically insignificant then the model for explaining EE is irrelevant. The significance of the explanatory variables is examined using the t-test.

Table 4.8 *Estimated Coefficients for the Cobb-Douglas Cost Frontiers*

	ESTIMATION 1 1980-1996		ESTIMATION 2 1986-1996	
VARIABLE	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	2.907*	20.572	2.862*	14.654
Output (Q)	0.466*	13.254	0.489*	10.385
Price[a] of				
Grains and Concentrates (PGC)	0.171*	7.330	0.118*	4.002
Hay and Forage (PHF)	-0.014	-1.025	0.008	0.482
Capital (PK)	0.336*	11.603	0.347*	9.757
Other Inputs (POI)	0.396*	10.993	0.412*	9.465
Year (YR)	0.012*	5.027	0.018*	5.587
σ	0.119*	11.220	0.313*	4.829
$\gamma\ (=\sigma^2_u/\sigma^2_u+\sigma^2_v)$	0.865*	35.934	0.950*	83.452

[a] Prices are relative to the price of labor

* Coefficients are significant at 5% percent.

Table 4.9 *Estimated Coefficients for the Simplified Translog Cost Frontiers*

	ESTIMATION 1 1980-1996		ESTIMATION 2 1986-1996	
VARIABLE	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	2.812*	12.555	2.338*	5.747
Output (Q)	0.487*	8.700	0.561*	5.689
Price[a] of				
Grains and Concentrates (PGC)	0.300*	6.080	0.209*	3.359
Hay and Forage (PHF)	-0.061*	-2.227	0.081*	2.245
Capital (PK)	0.398*	6.668	0.561*	5.929
Other Inputs (POI)	0.234*	3.390	-0.008	-0.075
Year (YR)	0.008	0.322	0.101	1.534
Q.YR	0.000	0.003	-0.010	-0.631
PGC.YR	-0.015*	-3.084	-0.014	-1.379
PHF.YR	0.005	1.734	-0.010	-1.776
PK.YR	-0.008	-1.408	-0.033*	-2.474
POI.YR	0.020*	2.825	0.063*	3.694
YR.YR	0.001	1.582	0.000	-0.254
σ	0.120*	9.882	0.137*	9.014
$\gamma\ (=\sigma^2_u/\sigma^2_u+\sigma^2_v)$	0.871*	33.807	0.891*	46.202

[a] Prices are relative to the price of labor.

* Coefficients are significant at 5% percent.

Table 4.10 *Likelihood Ratio (LR) Tests of Hypotheses for Parameters of the Cobb-Douglas and Simplified Translog Stochastic Cost Frontiers*

		COBB-DOUGLAS		SIMPLIFIED TRANSLOG	
HYPOTHESIS		Estimation 1 (1980-96)	Estimation 2 (1986-96)	Estimation 1 (1980-96)	Estimation 2 (1986-96)
a) H_0:γ=0					
Estimated frontier not different from OLS (average response) function.	LLF_U[a]	**250.83**	**171.55**	**260.53**	**190.94**
	LLF_R[a]	169.16	89.17	178.02	107.97
	LR[b]	163.33	164.76	165.03	165.95
	Critical Value (5% Level)	14.85*	13.4*	14.85*	13.4*
	Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0
b)(i)H_0:β_{ij}=0 i≤j=1,2,…,6					
(ii) H_0:β_{ij}=0 i≤j=1,2,…,5	LLF_U[a]	**285.51**	**214.74**	**285.51**	**214.74**
Interaction terms of the conventional translog are all equal to zero, implying (i) CD and (ii) Simplified translog*	LLF_R[a]	250.83	171.55	260.53	190.94
	LR[b]	69.36	86.38	49.96	47.6
	Critical Value (5% Level)	32.67*	22.36*	32.67*	22.36*
	Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0
c) (i) H_0:δ_1=δ_2=…=δ_5=0 (Estimation 1)	LLF_U[a]	**250.83**	**171.55**	**260.53**	**190.94**
(ii) H_0:δ_1=δ_2=…=δ_6=0 (Estimation 2)	LLF_R[a]	239.44	153.03	252.76	173.51
All parameters on the variables explaining technical efficiency are simultaneously equal to zero (i.e., no EE effects)	LR[b]	22.78	37.04	15.54	34.86
	Critical Value (5% Level)	11.07*	12.59*	11.07*	12.59*
	Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0

* Critical Values are obtained from Kodde and Palm (1986). These values entail a mixed χ^2 distribution. Because γ=0 lies on the boundary of the parameter space for γ, the LR statistic for testing if H_0: γ=0 is true has asymptotic distribution that is a mixture of Chi-square distributions (Coelli et al. 1998).

*The hypothesis whether the interaction terms with time of the simplified translog are all equal to zero (i.e., **H_0:β_{it}=0, i=1,2,…,6**) was rejected in Estimation 1 (1980-96) and was not rejected in Estimation 2 (1986-96).

[a] LLF_U and LLF_R are the log likelihood function values of the unrestricted and the restricted function, respectively.

[b] LR is the computed Likelihood Ratio value

Table 4.11 *Economic Efficiency (EE) Measures for Cobb-Douglas and Simplified Translog Cost Frontiers*

	COBB-DOUGLAS		SIMPLIFIED TRANSLOG	
	Estimation 1 (1980-96)	Estimation 2 (1986-96)	Estimation 1 (1980-96)	Estimation 2 (1986-96)
Mean EE	0.83	0.83	0.84	0.84
Median	0.86	0.86	0.87	0.87
Max. Value	0.97	0.97	0.97	0.97
Min. Value	0.32	0.32	0.33	0.31
Variance	0.011	0.013	0.11	0.012

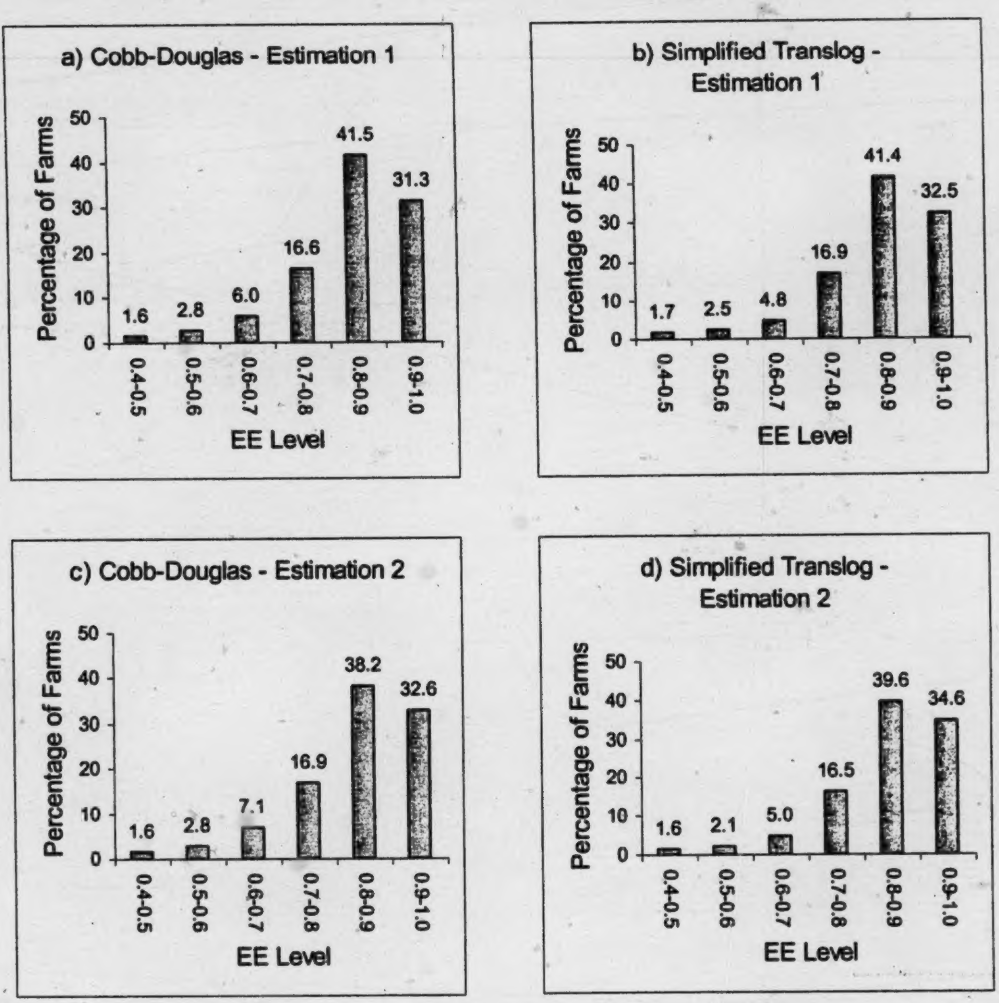

Figure 4.2 Distribution of Economic Efficiency (EE) in Percentage Terms, by Functional Form and Estimation

The null hypothesis for testing for the significance of the EE model was specified as in the production frontier,

$H_0: \xi_i = 0, \forall_i, i=1,2, \ldots, 5$ for Estimation 1 or i = 1, 2,..., 6 for Estimation 2[57];

Shouldn't this coefficient be ξ instead of δ?

that is, the coefficients on the variables explaining economic efficiency in the EE model are all equal to zero.

The LR test used values of the loglikelihood functions for stochastic cost frontiers estimated simultaneously with the full EE model (LLF_U) and the corresponding values for the frontiers when estimated with the EE model including only the constant term (LLF_R). From the results for this test, the hypothesis was rejected for all estimations (Table 4.10 -Hypothesis c). This implies that the EE model (Table 4.12) has statistical merit in modelling economic efficiency for Alberta dairy farmers.

Most of the results of the EE model are consistent with those of the TE model, at least for the coefficients that are statistically significant. One exception is the coefficient on breeding and veterinary services (BE). It was expected that factors that influence TE should also influence EE, since TE is a component of EE. Below is the elaboration of how the factors influence economic efficiency.

4.3.5.1 Ratio of Grain and Concentrates to Hay and Forage:

The coefficient for the ratio of grain and concentrates to hay and forage (GH), although statistically insignificant, is negative in three out of four estimations. The negative sign on the coefficient indicates that farmers would enhance economic efficiency by increasing the ratio, as was found in the case of technical efficiency. However, because the coefficient is statistically insignificant in most of the estimations, it remains inconclusive as to how the efficient allocation of inputs is influenced by the feed ingredient mix.

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[57] The variables in Estimation 1 include the ratio of Grains and Concentrates to Hay and Forage (GH), the ratio of Capital to Labour (KL), Breeding and Veterinary services (BE), Time variable (YR), and Herd size (HS). Estimation 2 includes Years of Farming (YF) in addition to the variables included in Estimation 1.

4.3.5.2 Capital to Labour Ratio

The coefficient for the capital-to-labour ratio variable (KL) is negative and highly statistically significant at a 5% level in all estimations. This implies that farms utilising more capital relative to labour tend to be more economically efficient. In other words, capital facilitates efficiency in cost minimisation to a greater degree than labour does for a given level of production.

4.3.5.3 Breeding and Veterinary Services:

The coefficient for the breeding and veterinary services (BE) variable is highly significant and positive in all estimations. This suggests that higher breeding and veterinary services per cow do not tend to translate into greater economic efficiency for farms. These results are opposite of the results for the TE model, and are contrary to the hypothesized relationship. The implication is that though more utilisation of breeding and veterinary services tends to enhance output, it may not simultaneously be resulting in optimal allocation of resources; hence, it does not facilitate the minimisation of production costs.

4.3.5.4 Trend Variable:

The coefficient on the trend variable (YR) is again statistically insignificant for all estimations. These results indicate that economic efficiency does not change significantly over time. Hence, time seems to have no bearing on economic efficiency, as was similarly indicated for technical efficiency in the production frontier model.

Table 4.12 *Coefficient Estimates for Models Explaining Economic Efficiency of a Sample of Alberta Dairy Farms*

	COBB-DOUGLAS				SIMPLIFIED TRANSLOG			
	Estimation 1 (1980-96)		Estimation 2 (1986-96)		Estimation 1 (1980-96)		Estimation 2 (1986-96)	
VARIABLE[+]	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO
CONST.	-0.285*	-2.880	-2.292*	-3.606	-0.313*	-3.144	-0.686*	-4.583
GH	-0.042	-1.168	-0.029	-0.577	-0.042	-1.168	0.024	0.581
KL	-0.232*	-2.660	-0.407*	-12.831	-0.231*	-2.052	-0.307*	-3.340
BE	0.004*	4.255	0.012*	4.871	0.005*	4.125	0.007*	5.084
YR	-0.005	-0.769	-0.027	-1.813	-0.004	-0.545	-0.032*	-2.745
YF	-	-	0.028*	4.470	-	-	0.013*	5.496
HS	0.003*	4.069	0.005*	3.421	0.002*	3.172	0.003*	3.565

*Coefficient is significant at 5% level.

[+] The abbreviations for the variables are as follows
GH – the ratio of Grains and Concentrates to Hay and Forage; KL – the ratio of Capital to Labour; BE – Breeding and Veterinary services; YR – Time trend; YF – Years of Farming; HS – Herd size.

4.3.5.5 Years of Farming:

The coefficient for the years of farming variable (YF) is significant and positive in all estimations, indicating that farmers with fewer years of farming tend to be more economically efficient. These results are similar to those for the TE model. However, they are contrary to the hypothesis that efficiency improves with time because of learning. As conjectured in the discussion of results of the TE model, farmers with few years of experience are likely to be young, and perhaps more "dynamic" than more experienced producers.

4.3.5.6 Herd Size:

The coefficient for herd size (HS) is positive and significant in all estimations, which indicates that smaller herds tend to be more economically efficient than larger herds. These results do not tend to support the observed trend that show that herd expansion is a viable strategy. At the very least, this would suggest that any benefits of size economies in Alberta dairy production may be partially offset by reduced efficiency. This may be rationalized in view of the possibility that smaller herds allow for better management and thus better decision making, or due to differences in cow quality in terms of genetics, either of which would result in improved economic efficiency.

4.3.6 Economic Efficiency and Allocative Efficiency

An attempt was made to decompose economic efficiency into its two components of allocative and technical efficiency, following the method suggested by Kopp and Diewert (1982) (Section 3.2.4). However, for both the simplified translog and translog formulations there was no convergence to a solution. This method involves computations for a system of equations to solve for relative input prices and input amounts used at the point that is technically efficient but allocatively inefficient.

Moreover, similar computations for the Cobb-Douglas formulation gave results that implied that efficiency was greater than one, which was at variance with measures of technical and economic efficiency in the single-equation estimations. For these reasons the Kopp and Diewert method could not be pursued further.

The breakdown of economic efficiency into technical and allocative efficiency was derived mathematically only for the Cobb-Douglas formulation, since it is self-dual[58]. Mean allocative efficiency (Mean AE) was derived following the relationship that economic efficiency is a product of allocative and technical efficiency. First, the EE estimate from the cost frontier for each farm was divided by the corresponding mean TE estimate from the production frontier. Mean AE was then obtained as a simple average of AE's for individual farms. From this computation, average allocative efficiency was found to equal 0.88 percent and 0.92 percent for Estimation 1 and Estimation 2, respectively. This implies that the average farm would reduce its cost by 8 to 12 percent if it were to allocate the inputs in an optimal fashion, according to their relative prices.

Figure 4.3 shows the percentage distribution of AE levels for Estimation 1 and Estimation 2 computed from corresponding Cobb-Douglas production and cost frontiers. Approximately 85 percent of the farms are operating at an allocative efficiency level that is 80 percent or higher, and 93 percent of the farms are at a 70 percent level of efficiency or higher. As in the case of TE and EE, most farms are in the 70-99 percent range of allocative efficiency, which is further indication of a high degree of homogeneity of performance among Alberta herds.

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[58] As discussed in the previous chapter, the results of the decomposition procedure used here should be interpreted with caution. The usual process for utilizing the self-dual nature of the Cobb-Douglas functional form would involve deriving the cost frontier directly from the production frontier. In the current study, the cost frontier is estimated separately from the production frontier. Hence, there were no restrictions imposed that would guarantee consistency in parameter estimates.

4.4 Discussion of Results

The results presented in the two sub-sections above are further analysed on two levels. The first level is the relative performance among Alberta farms. The second level is the comparison with other studies on efficiency in North America. However, comparison at the second level should be taken with caution because it can only be justified if, at least, the technology used by the farms, the methods of estimation, and the variables included in the used models and their definitions are the same. For example, Bravo-Ureta and Rieger (1990) found considerable variability in technical efficiency of farms estimated with different methods, even though the data set was the same. However, the results showed technical efficiency measures from these methods to be highly correlated, giving similar ranking of farms. Cloutier and Rowley (1993) used a deterministic model to examine the effect of linear aggregation of inputs (by value). They found that the number of fully efficient farms as well as the value of TE decreased with increased aggregation, despite using a common set of data.

4.4.1 Efficiency Levels

The findings suggest that both average technical and economic efficiency levels of Alberta dairy farms are high (for example, mean TE is 0.91 and mean EE is 0.83). In addition, the findings suggest that the difference between the highest and lowest efficiency levels is bigger with respect to economic efficiency than for technical efficiency. These findings are somewhat similar to those of previous studies that analysed efficiency of Alberta dairy farmers. The mean TE and mean EE levels in this study are slightly lower than in Richards and Jeffrey (2000) (0.94 for TE and 0.91 for EE) and higher than the mean TE in Richards and Jeffrey (1996) (0.85 with iterative Average Frontier (AF) method or 0.83 with Composed Error (CE) method) for the same farms.

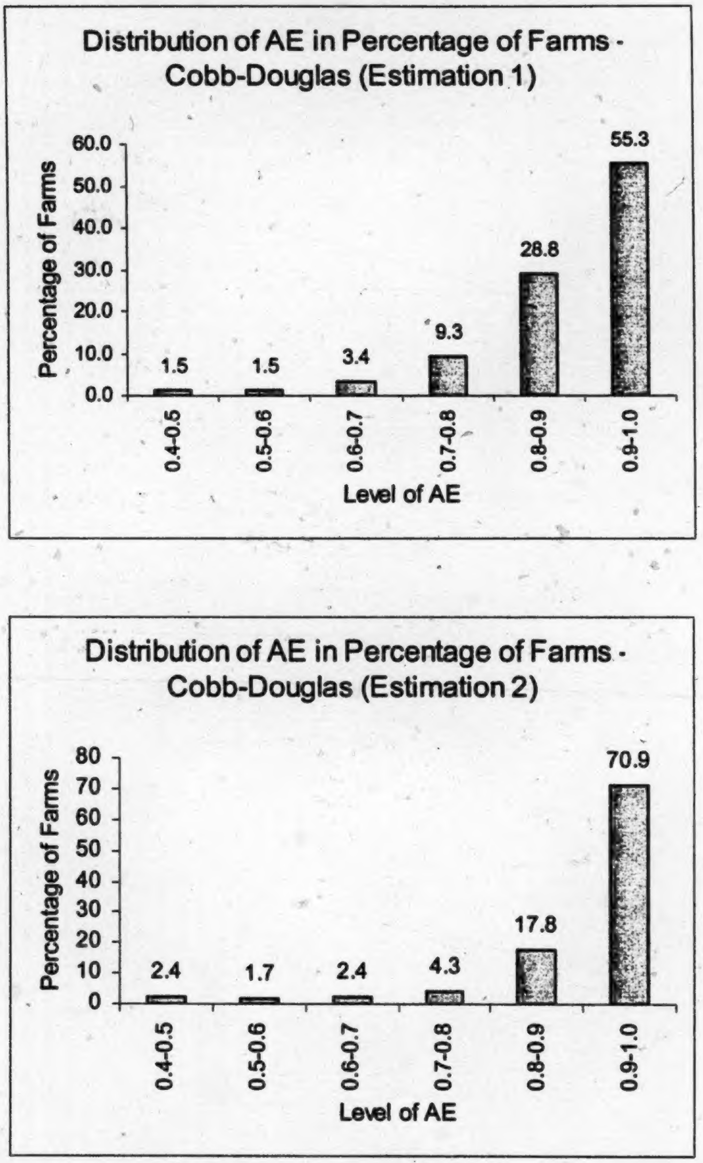

Figure 4.3 Distribution of Allocative Efficiency (AE) (Percentage of Farms) for Cobb-Douglas Frontiers, by Estimation

Although the studies drew from the same data pool, there are differences in the methods used and in the results that reveal the contribution by this study. In addition to using different methods in estimating efficiency, the previous studies relied on short periods; hence, they did not analyse efficiency over time. By analysing efficiency over a long period, this study has shed more light on the performance of Alberta farms in that the effect of time on both technical and economic efficiency levels has been tested and found to be statistically insignificant.

The comparison of results with other Canadian and US studies can only signal the similarity or differences in the measures, since the studies involved are based on different data sets and methods. Weersink et al. (1990) results for Ontario dairy farms (0.92 for overall efficiency average and 0.95 for mean TE) are slightly higher than the estimates from this study. Mbaga et al. (2000) results for Quebec dairy farms with SFA and DEA methods are higher (mean TE ranges between 0.92 to 0.97 for SFA and is 0.92 and 0.95 for DEA). The efficiency levels in this study are closer to those in Cloutier and Rowley (1993) for Quebec dairy farms (TE of 0.91 for 1989 and 0.88 for 1988).

Moreover, in terms of the distribution of efficiency levels, Weersink et al. (1990) found that more than 90 percent of the farms performed above 90 percent level of TE as compared to 65 percent of the farms in this study. Mbaga et al (2000) found that with DEA, 66 percent of the farms were within that range, whereas with SFA for the Generalized Leontief frontiers, 93 percent of the farms were within that range. Romain and Lambert (1992) found a smaller range in cost of production between the most efficient and the least efficient farms for Quebec and Ontario dairy farms (13% and 16% lower, respectively) than that found by this study (65% lower for EE). When compared to US studies (e.g., Bravo-Ureta, 1986; Kumbhakar et al., 1989; Tauer, 1993; Ahmad & Bravo-Ureta, 1996), the findings of this study on estimated efficiency are higher.

4.4.2 Functional Forms

The choice of functional form is another contribution by this study to the understanding of Alberta dairy farms, since previous studies on Alberta dairy production (e.g., Richards & Jeffrey 1996; 1998; 2000) did not address this issue. The results of the Cobb-Douglas and simplified translog formulations seem to be better with regard to the sign and statistical significance of the coefficients as compared to the conventional translog formulation. Still the translog formulation was found to be the best representation of the data, as it showed the highest likelihood from which the used data could have been drawn, in addition to that the hypothesis test as to whether its interaction terms are irrelevant could not be rejected.

The implications of these results for the choice of functional form are two-fold. First, the results indicate that the interaction terms of the translog and, to a lesser extent, those of simplified translog are important in modelling the structure of production and costs of production for milk output by Alberta farms. The interaction terms add flexibility to the examined structure by imposing fewer restrictions on the hypothesized relationships. Fewer restrictions allow the model to more closely approximate the true underlying technical or economic relationships. However, as discussed earlier, the results of the conventional translog formulation could not meet a priori expectations; for example, most of its parameter estimates are statistically insignificant. Second, even though both the Cobb-Douglas and simplified translog formulations are not as flexible as the translog formulation, the results of these formulations were consistent with a priori expectations. In the view of this study, correct economic interpretation should be given pre-eminence over structural elegance. Hence, the results of Cobb-Douglas and simplified translog were adopted, with caution because of the restrictions being imposed (as discussed earlier).

Empirical evidence from other North American studies as to which formulation is the most appropriate is not consistent. Kumbhakar et al. (1991) found that the Cobb-Douglas formulation was not as appropriate as the Zellner-Revankar specification. TE measures estimated by Ahmad and Bravo-Ureta (1996) were invariant as to the Cobb-Douglas or the simplified translog formulations. Mbaga et al. (2000) found the Generalized Leontief to be the most appropriate formulation in the non-maize region, but the Cobb-Douglas formulation the most appropriate in the maize region. Thus, the findings in these studies, in addition to those of this study, tend to imply that the appropriateness of functional forms is dependent on the data used.

4.4.3 Herd Size

In this study, smaller herds were found to be associated with high levels of economic efficiency and vice versa for larger herds. However, the relationship with respect to technical efficiency was found to be inconclusive. The results with respect to economic efficiency tend to indicate that the strategy calling for herd size expansion in preparation for increased competition (Western Producer, 1999) may come at the cost of reduced economic efficiency. Many U.S. studies have consistently reported a positive effect of herd size expansion on efficiency, although evidence from Canadian studies with regard to this aspect is mixed. The results of this study, as are the results of related Canadian studies, may be influenced by the supply management regime that has been in operation during the time period for the current study. Policies related to the supply management system (e.g., farm-level marketing quotas) will undoubtedly have had a bearing on the level (as differentiated from the degree) of efficiency of all farmers, including those with best practice.

As well, the existence of the quota system has placed an additional constraint that limits producers' ability to expand their herds. Thus the difference in herd size efficiency effects that exist between U.S. and Canadian studies may simply be due to a lack of Canadian herds of sufficient size to truly capture a positive herd size effect. Hence, the results of the current study may not be relevant in a new regime of free trade and increased international competition.

With respect to the supply management regime, the findings of this study with regard to economic efficiency may be interpreted in two ways. First, although large farms may be able to exploit economies of scale, this advantage is generally at the expense of reduced intensity and attention to the cow. Thus, output gains from economies of scale may be outweighed by productivity loss due to reduced care. Second, the results need to be qualified in that herd size was included in the models as if all herds were homogeneous, without due consideration to quality differences in terms of genetics, which are captured in technical change. Since technical change was found to be significant, part of this is likely to be reflected in the improved quality of the herds over time, hence the likelihood that a smaller number of improved head may be equally productive or same number of improved head may be more productive.

The above results are not similar to Jeffrey and Richards (2000) who found for Alberta farms that technical efficiency and allocative efficiency appear to be very similar among herd size groups. The divergence in findings may be explained in that Jeffrey and Richards (2000) used BCA indexes to account for genetic quality differences in herds.

Evidence from other Canadian studies is mixed, as already pointed out. Romain and Lambert (1992) found that farm size has no bearing on cash costs and costs of production in general. Weersink et al. (1990) found that TE tended to rise with herd size. Cloutier and Rowley (1990) found that the influence of herd size on TE varied, depending on how the variable was used in the estimations. When herds were not categorised, the results indicated that small herds were associated with higher levels of

TE than large ones. However, when the herds were categorised into small and large ones and separate estimations performed, the opposite was the case.

Some studies have estimated the maximum efficiency herd size. Weersink et al. (1990) estimated maximum TE herd size to be 102 head for Ontario farms and Richards and Jeffrey (2000) estimated maximum EE herd size at 70 head for Alberta. The maximum EE herd size that Richards and Jeffrey (2000) estimated is lower than the present average herd size of approximately 101 cows (Statistics Canada, 2002)[59]. Hence, the results of this study, in addition to previous ones, imply that potential gains in efficiency through herd expansion for Alberta farms may be limited. It should, however, be recognized (and noted) that this result may be somewhat affected by the state of current milking and housing technology that is employed by producers in the sample. It may well be the case that if the sample included producers who were much larger in herd size, we might see some different results because of increased frequency of use of alternative technologies that are better suited (i.e., conducive to increased efficiency) for very large herds.

Most studies on efficiency in US dairy production show a positive relationship between efficiency and herd size (e.g., Tauer & Belbase, 1987; Bravo-Ureta & Rieger, 1990, 1991; Tauer, 1993). However, the study by Bravo-Ureta (1986) found that technical efficiency of farms was statistically independent of the size of the farm, as measured by the number of cows.

4.4.4 Capital to Labour Ratio

The results indicated a positive relationship between the capital-to-labour ratio and economic efficiency. This implies that increased capital intensity is more important in enhancing cost minimisation than is increased labour intensity. Although the relationship between the ratio of capital to labour and technical efficiency was not conclusive, the findings of this study with respect to economic efficiency do not share commonality with findings for at least some other studies. Jeffrey and Richards (2000) found that labour quality contributes significantly to economic performance; Romain and Lambert (1994)[60] found that technical efficiency is directly related to the degree of labour intensity; and Weersink et al. (1990) found highly capitalised farms to be less technically efficient. Whereas these previous results tend to suggest that, given the amounts of capital and labour that farms are now using, intensifying labour relative to capital is likely to lead to increased output of milk or to reduced costs of producing milk, the results of this study tend to suggest the opposite.

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[59] Quoted from a table summarizing the number of dairy cows by province as of January, 2002; available **http://www.dairyinfo.agr.ca/cdicfpfarms.htm.**

4.4.5 Breeding and Veterinary Services

The study found contradictory results for breeding and veterinary services. Higher breeding and veterinary expenses are associated with higher levels of technical efficiency and lower levels of economic efficiency. This implies that the positive effect of breeding and veterinary services on technical efficiency is outweighed by its negative effect on allocative efficiency. The findings are not consistent with those of Richards and Jeffrey (2000), who found that breeding and veterinary services tend to enhance economic performance. The results, however, are consistent with the findings of Romain and Lambert (1994) for both Ontario and Quebec[61]. The variable in this study included amounts spent on breeding and veterinary expense per cow. This includes expenditures to treat the herd if there is a health problem, to purchase breeding services through artificial insemination, and to pay for other attention to herd health (Richards & Jeffrey, 1998). Therefore, the influence of the variable may have spill over of technological change, but no attempt was made to disentangle it from efficiency effects. In spite of lack of preciseness in valuing the variable, the implication of the findings that farmers may improve their technical performance through improving the quality of their breeding programmes is still valid.

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[60] The Romain and Lambert (1994) study is referred to in Richards and Jeffrey (2000).

[61] The results of Romain and Lambert (1994) are referred to in Richards and Jeffrey (2000).

4.4.6 Ratio of Grains and Concentrates to Hay and Forage

The estimated coefficient for ratio of grains and concentrates to hay and forage (GH) indicated that EE is enhanced by increases in the ratio, whereas the relationship with technical efficiency was statistically insignificant. In spite of being more expensive per unit, grains and concentrates have a higher nutritional content than hay and forage. Depending on the current level grains and concentrates being fed, their increased relative use enhance both technical and allocative efficiency. The results are consistent with Richards and Jeffrey (2000) findings that utilized the same data set, though for a shorter period of study. Romain and Lambert's (1994)[62] study found that the increases in the ratio tended to enhance technical efficiency.

4.4.7 Scale of Operation

The results for the production frontier analysis indicate that the Alberta farmers are operating within stage II of the milk production process, with decreasing returns to scale (DRS) and positive marginal products from all inputs used[63]. Evidence from other studies is mixed. Richards and Jeffrey (2000) found that CRS could not be rejected for Alberta farms; Kumbhakar et al. (1991) found DRS for US dairy farms; Tauer and Belbase (1987) and Tauer (1993) for New York farms and Ahmad and Bravo-Ureta (1996) found IRS for Vermont dairy farms.

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[62] These findings are also referred to in Richards and Jeffery (2000).

[63] As noted earlier in this chapter, the overall results from this study with respect to returns to scale or size are conflicting. While the production frontier estimates suggest decreasing returns to scale, the results from the cost frontier analysis suggest the reverse case. It is unclear as to what is causing this inconsistency.

4.5 Effect of Alternative Distributional Assumptions for (u_i) on Efficiency Measures

This section summarizes results from the estimation of production and cost frontiers, while examining to the sensitivity of efficiency estimates to assumptions concerning the distribution of the inefficiency term (u_i). This analysis is carried out using the full data set (i.e., data used earlier in Estimation 1). The first sub-section (4.5.1) reports and discusses results from the estimation of production frontiers and technical efficiency (TE) with respect to the exponential, half-normal, and truncated-normal distributions in Cobb-Douglas and (conventional) translog production frontier specifications. The second sub-section (4.5.2) reports and discusses results from the estimation of cost frontiers and economic efficiency (EE) for both the Cobb-Douglas and translog cost frontier specifications. However, results from translog cost frontiers are only with respect to the half-normal and truncated normal distributions, as the parameters estimated for the exponential distribution assumption failed to converge at their maximum.

The previous discussion pointed out problems with the results from the translog specification in the additional analysis. However, it was decided to retain this functional form for use in the u_i distribution analysis. The previous results were related to the assumption of the truncated distribution for u_i only. It was therefore important to use the translog specification, if only to ascertain that the problems encountered were not specific to a particular distribution. Moreover, the translog specification was needed in testing for the adequacy of the Cobb-Douglas in relation to several distribution assumptions, so as to further gauge how the assumed distribution of u_i impacts on the choice of functional form.

4.5.1 Production Frontier Results

In this sub-section, the results of production frontier estimations for the three distributions (i.e., exponential, half-normal, and truncated-normal) are presented and discussed. They include, for both the Cobb-Douglas and translog frontiers, coefficient estimates for the parameters of the frontiers (4.5.1.1), elasticities of output with respect to inputs (4.5.1.2), technical efficiency (TE) measures (4.5.1.3), correlation coefficients for pairs of efficiency estimates (4.5.1.4), and the LR test for the equality of production frontier specifications across distributions (4.5.1.5).

4.5.1.1 Production Frontier Parameter Estimates

The coefficient estimates for the parameters of the Cobb-Douglas and translog frontiers are presented in Tables 4.13 and 4.14, respectively. In the Cobb-Douglas specification (Table 4.13), the coefficients for all variables except hay and forage (HF) are statistically significant under all three distributional assumptions. The coefficients for each variable across the distributions are almost of equal magnitude, which implies some consistency for the estimated frontiers. In addition, all of the coefficients, which are elasticities of output with respect to the inputs, are positive and hence are consistent with a priori theoretical expectations.

The coefficient estimates for the parameters of the translog specification do not themselves have an economic interpretation. However, they may be used to derive the elasticities of output with respect to inputs. Their statistical significance indicates the statistical significance of the estimated frontiers.

As Table 4.14 shows, most of the coefficients of the translog specification are not statistically significant, and some vary significantly in magnitude across distributional assumptions. Whereas the coefficients for capital (K) and grains and concentrates (GC) are not statistically significant under some distributional assumptions, the coefficient on labour (L) is not statistically significant for any of the three distributions. Moreover, most coefficients on interaction terms are not statistically significant, except (mainly) those for grains and concentrates (GC) and capital (K). The coefficients on some variables vary in magnitude by a large margin across distributions, which may indicate a significant difference in location for the frontiers. For example, the coefficients on other inputs (OI) range from 0.46 for the exponential distribution to 0.73 for the truncated-normal distribution.

4.5.1.2 Output Elasticities with Respect to Inputs, Returns to Scale and Technical Change

Table 4.15 summarizes the estimation results in terms of output elasticities with respect to inputs, returns to scale (RTS) and technical change for the three inefficiency error term distributions. The measures are similar across the three distributions of u_i for the Cobb-Douglas frontiers, but differ substantially for the translog frontiers. The exception is technical change, which is almost identical across all of the distributions (0.013-0.014) for both the Cobb-Douglas and translog specifications.

For the Cobb-Douglas frontiers, all of the elasticities of output are positive, implying that increased use of any of the inputs will lead to increased output. The magnitudes of the elasticities differ between inputs, ranging from 0.004 for hay and forage (HF) to 0.146 for other inputs (OI). However, they vary slightly and inconsistently across distributions. Overall, the variations in individual elasticities tend to cancel out such that they sum to almost the same values of RTS (0.33-0.34), implying decreasing returns to scale (DRS). In addition to the small variations in the magnitudes of the elasticities, the consistency in RTS values and rates of technical change suggest that the distribution of u_i has an insignificant effect on the estimated Cobb-Douglas frontiers. The output elasticities with respect to inputs for the translog frontiers, calculated using mean values of the variables, include some that are negative. However, across the three distributions the sign on each of the elasticities is the same. The elasticities with respect to other inputs (OI) and with respect to hay and forage (HF) are positive. The rest of the elasticities are negative, an indication of a serious problem with estimations of the translog frontier, since the negative elasticities imply that the related inputs are over-utilized, with the effect that increased use of any of those inputs would lead to a decrease in output.

For the exponential and truncated normal distributions, this negative effect on output is compensated for by the positive effect such that the value of returns to scale is slightly above zero (0.028 and 0.086, respectively). For the half-normal distribution however, the negative effect on output outweighs the positive effect such that the value of returns to scale is slightly below zero (-0.064). The negative value of RTS does not make economic sense, since it implies that a proportionate increase in all inputs would lead to a decrease in output.

On average, the producers will be producing in stage III of the production process, which no rational producers would do because they could produce the same output by utilizing a lesser amount of inputs.

The above measures are not easily comparable across the two functional specifications, except for technical change. For example, the values for the elasticity with respect to hay and forage (HF), which are the smallest for the Cobb-Douglas frontiers, are relatively larger for the translog frontiers; the values for the elasticity with respect to other inputs (OI) that are the largest in Cobb-Douglas frontiers are relatively small when compared to corresponding values for the translog frontiers. Yet, the values for returns to scale (RTS) for the Cobb-Douglas frontiers are relatively large when compared to the corresponding values for the translog frontiers.

Table 4.13 *Coefficient Estimates for the Parameters of Cobb-Douglas Production Frontiers, by Alternative Distribution Assumption for u_i*

	EXPONENTIAL		HALF-NORMAL		TRUNCATED-NORMAL	
VARIABLE	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	3.072*	47.570	3.088*	46.395	3.089*	46.195
Grains and concentrates (GC)	0.113*	13.256	0.113*	13.406	0.118*	13.864
Hay and forage (HF)	0.005	0.534	0.008	0.764	0.004	0.384
Labor (L)	0.030*	4.021	0.031*	3.911	0.027*	3.398
Capital (K)	0.043*	2.612	0.043*	2.524	0.049*	2.902
Other inputs (OI)	0.141*	8.703	0.146*	8.777	0.136*	8.134
Year (YR)	0.014*	9.467	0.013*	8.769	0.014*	9.256
θ	10.702*	13.523			-	-
σ_v	0.118*	40.737			-	-
λ	-	-	1.502*	13.194	1.679	0.118
σ	-	-	0.201*	30.475	0.226	0.004
μ/σ_u	-	-	-	-	0.629*	3.089

*Coefficient is significant at 5% level.

Table 4.14 ***Coefficient Estimates for the Parameters of Translog Production Frontiers, by Alternative Distribution Assumption for u_i***

	EXPONENTIAL		HALF-NORMAL		TRUNCATED-NORMAL	
VARIABLE	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	3.681*	5.401	3.951*	5.482	3.586*	5.050
Grains and concentrates (GC)	-0.276	-1.259	-0.412	-1.753	-0.520*	-2.275
Hay and forage (HF)	0.339*	2.372	0.305	1.973	0.354*	2.345
Labor (L)	-0.126	-0.977	-0.172	-1.272	-0.147	-1.095
Capital (K)	-0.517	-1.973	-0.571	-2.059	-0.443	-1.641
Other inputs (OI)	0.458*	2.063	0.603	2.620	0.726*	3.222
Year (YR)	0.064*	9.467	0.058*	2.456	0.053*	2.275
GC.GC	0.063	1.640	0.064	1.554	0.074	1.949
GC.HF	0.011	0.386	0.010	0.355	0.023	0.817
GC.L	-0.003	-0.124	0.019	0.648	0.023	0.775
GC.K	0.245*	5.049	0.242*	4.818	0.248*	5.081
GC.OI	-0.190*	-3.959	-0.176*	-3.598	-0.174*	-3.667
GC.YR	0.004	0.992	0.003	0.713	0.004	0.807
HF.HF	-0.082*	-3.395	-0.073*	-2.720	-0.073*	-2.769
HF.L	0.026	1.645	0.028	1.643	0.024	1.472
HF.K	-0.069*	-2.307	-0.061	-1.878	-0.081*	-2.518
HF.OI	-0.013	-0.373	-0.020	-0.527	-0.023	-0.626
HF.YR	0.001	0.449	0.001	0.437	0.002	0.568
L.L	0.013	0.678	0.015	0.752	0.013	0.699
L.K	-0.001	-0.023	0.005	0.178	-0.003	-0.111
L.OI	0.027	0.926	0.009	0.275	0.010	0.327
L.YR	-0.005*	-1.969	-0.004	-1.615	-0.004	-1.668
K.K	0.001	0.017	-0.001	-0.019	-0.014	-0.212
K.OI	0.074	1.428	0.077	1.420	0.078	1.486
K.YR	-0.025*	-5.703	-0.024*	-5.361	-0.025*	-5.773
OI.OI	-0.059	-0.702	-0.088	-0.988	-0.136	-1.550
OI.YR	0.013*	1.967	0.014*	2.024	0.017*	2.495
θ	-0.001	-0.793	-0.001	-0.733	-0.001	-0.952
σ_v	0.105	21.181				
λ			1.861	8.229	2.178	6.669
σ			0.204	27.707	0.239	6.241
μ/σ_u					0.689	0.745

*Coefficient is significant at 5% level.

Table 4.15 *Output Elasticities, Returns to Scale and Technical Change for Production Frontier Estimations, by Alternative Distribution Assumption for u_i* [a]

	COBB-DOUGLAS			TRANSLOG		
INPUT	EXPONEN -TIAL	HALF-NORMAL	TRUNCATED-NORMAL	EXPONEN -TIAL	HALF-NORMAL	TRUNCATED-NORMAL
Grains and Concentrates	0.113	0.113	0.118	-0.15	-0.253	-0.326
Hay and Forage	0.005	0.008	0.004	0.212	0.189	0.224
Labour	0.030	0.031	0.027	-0.064	-0.096	-0.08
Capital	0.043	0.043	0.049	-0.267	-0.309	-0.215
Other Inputs	0.141	0.146	0.136	0.297	0.405	0.481
Returns to Scale (RTS)	**0.331**	**0.341**	**0.334**	**0.028**	**-0.064**	**0.084**
Technical Change	0.014	0.013	0.014	0.013	0.013	0.014

[a] For the translog specification the elasticities were calculated at the mean values for the variables. Note that the elasticities in Table 4.14 differ slightly from those presented in Table 4.3. Whereas the results in Table 4.3 are based on maximizing the loglikelihood function in the Battese-Coelli (1996) model that uses γ for the variance parameter, (Eqn. 2.5), those in Table 4.14 are based on Aigner et al. (1977) that instead uses λ (Eqn. 2.7).

4.5.1.3 Technical Efficiency (TE) Measures

The average performance of Alberta dairy farmers with regard to alternative distributions of the inefficiency error term is reflected in the mean and median TE values (Table 4.16). The mean TE differs among alternative distributions by a small margin, ranging from 0.91 for the exponential distribution to 0.96 for the half-normal distribution. Mean values for the half-normal distribution are the highest in both the Cobb-Douglas and translog estimations (0.95 and 0.96, respectively), followed by the truncated normal (0.93 and 0.94), with the exponential distribution having the lowest mean TE (0.91).

The values of median TE for the half-normal and truncated normal distributions are the same as those for mean TE. This implies that 50 percent of the sampled farmers produced more output per unit of input than the "average" farmer, even with the observed mean TE being so high. For the exponential distribution, the median TE is higher than the mean TE (0.93 in both specifications), which indicates that even though it ranks the lowest with respect to average performance, more than 50 percent of farmers performed better than the "average" farmer.

The higher mean TE value for the half-normal distribution is reflected in the distribution of TE values (Fig. 4.3). Of the total farm sample, 53.5% in the case of the Cobb-Douglas and 68.1% in the case of the translog display 95% efficiency or better. The exponential distribution ranks second to the half-normal distribution in the percentage of farms with TE levels above 0.95 (18.8% in Cobb-Douglas and 20.5% in translog specifications), and it compares favourably to the truncated normal distribution (6.9% in Cobb-Douglas and 12.4% in translog specifications).

Table 4.16 *Technical Efficiency (TE) Measures for Cobb-Douglas and Translog Production Frontiers, by Alternative Distribution Assumption of u_i*

	COBB-DOUGLAS			TRANSLOG		
	EXPONENTIAL	HALF-NORMAL	TRUNCATED-NORMAL	EXPONENTIAL	HALF-NORMAL	TRUNCATED-NORMAL
Mean TE	0.91	0.95	0.93	0.91	0.96	0.94
Median TE	0.93	0.95	0.93	0.93	0.96	0.94
Max. Value	0.98	0.99	0.98	0.98	0.99	0.99
Min. Value	0.55	0.79	0.93	0.51	0.78	0.93
Variance	0.003	0.0004	0.0001	0.004	0.0004	0.0001

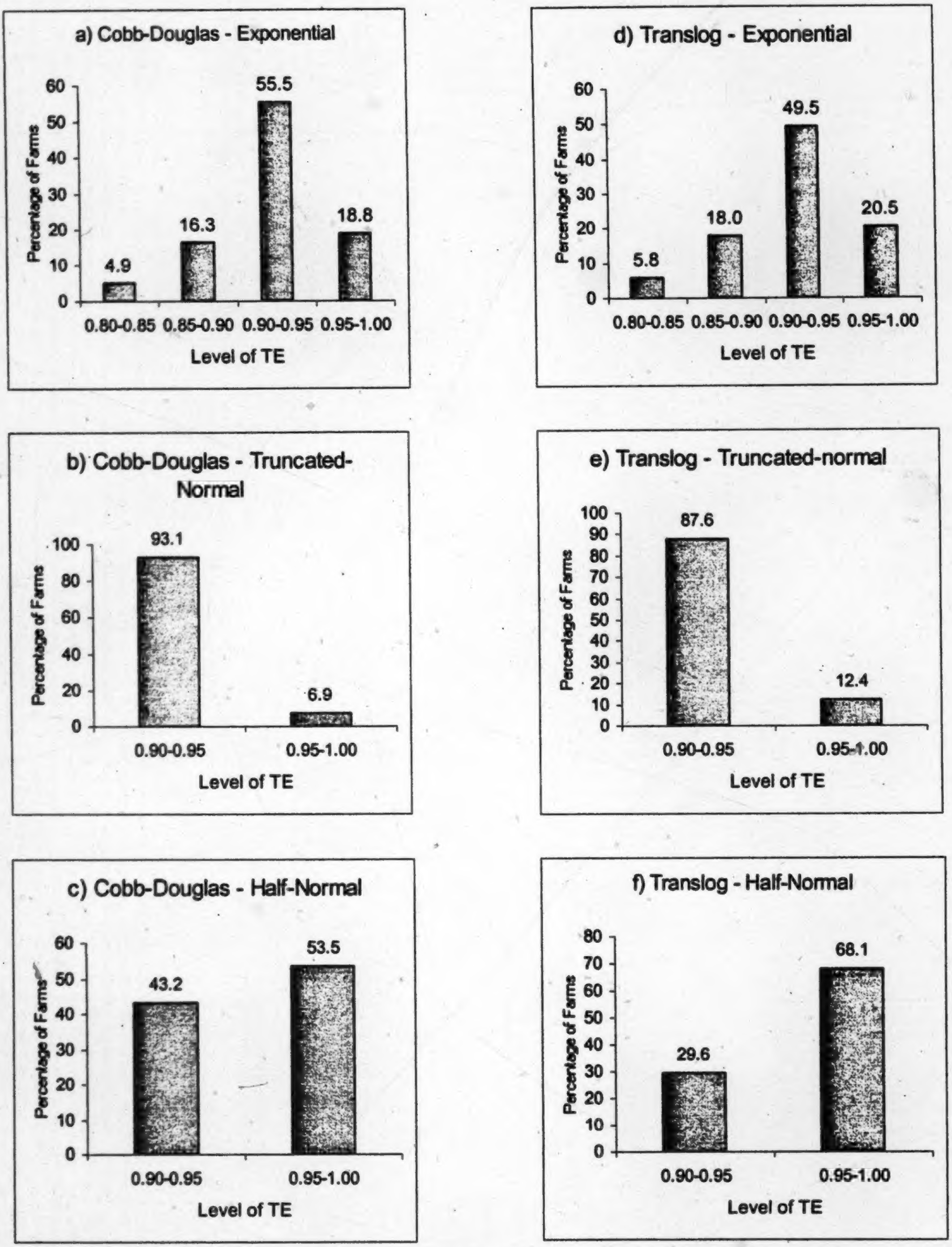

Figure 4.4 Distribution of Technical Efficiency (TE) in Percentage terms, by Alternative Distribution Assumption for u_i

In addition to high average TE measures, Table 4.16 illustrates that the variance between TE estimates is small, the largest variance being with the exponential distribution (0.003-0.004), which contrasts with the half-normal (0.0004) and truncated- normal (0.0001) distributions. The relatively larger variance for the exponential distribution is reflected in the distribution of individual TE measures for the distribution, which covers a wider range (0.51-0.98 in Cobb-Douglas and 0.55-0.98 in translog specifications) as compared to the half-normal (0.78-0.99) and truncated-normal (0.93-0.98) distributions. The small variance values among estimates, especially from the truncated-normal and half-normal distributions, point to a high degree of homogeneity among Alberta dairy products, as previously observed and discussed.

4.5.1.4 Correlation Coefficients of TE Estimates

The effect of the distribution of u_i on TE estimates was examined further using the pair-wise correlation coefficients of TE estimates, which are summarized in Table 4.17. All of the reported coefficients were found to be significantly different from zero at 1 % level. The discussed correlation coefficients are reported in the table's three sections. In the upper left section is the Cobb-Douglas form and in the bottom right section is the translog form, with varying distribution assumptions. In these two sections, the diagonal elements are correlation coefficients for the same functional form and same distributional assumptions, so all are equal to one. In the upper right section are the cross-functional form comparisons, with diagonal elements representing correlation coefficients for different functional forms (i.e., Cobb-Douglas versus translog) with the same assumed distribution. The coefficients from each particular functional specification show a variation in magnitude, which implies that the ranking of individual farms' TE differs between different distributions. For the Cobb-Douglas specification (upper left section), the correlation coefficient is larger and positive for the TE estimates of the half-normal and truncated-normal distributions (0.859).

Table 4.17 *Spearman's Rank Correlation Coefficients of Technical Efficiency (TE) Estimates from Cobb-Douglas and Translog Production Frontiers for Alternative Distribution Assumptions*

		COBB-DOUGLAS			TRANSLOG		
		EXPONENTIAL	TRUNCATED-NORMAL	HALF-NORMAL	EXPONENTIAL	TRUNCATED-NORMAL	HALF-NORMAL
COBB-DOUGLAS	EXPONENTIAL	1.000	0.210	0.097	0.692	0.760	0.692
	TRUNCATED-NORMAL		1.000	0.859	0.205	0.906	0.841
	HALF-NORMAL			1.000	0.106	0.760	0.889
TRANSLOG	EXPONENTIAL				1.000	0.254	0.397
	TRUNCATED-NORMAL					1.000	0.925
	HALF-NORMAL						1.000

The correlation coefficients for combination involving the exponential distribution are positive but very small (0.210 for exponential-truncated normal and 0.097 for the exponential- half normal). A similar pattern as in the Cobb-Douglas frontiers is observed in the translog frontiers for the exponential distribution (bottom right – 0.254 and 0.397 for truncated and half normal distributions, respectively). This pattern indicates that the levels of TE estimates by farm for the half-normal and truncated normal distributions are ranked more closely, whereas those for the exponential distribution are ranked differently from the other two.

The correlation coefficients of TE estimates for the same distribution across functional specifications are positive and relatively large, especially for the half-normal and truncated normal distributions. The TE estimates from the truncated-normal distribution are more highly correlated (0.906) than from the half-normal and exponential distributions (0.889 and 0.692, respectively). The relatively high correlation coefficients by distributions across functional specifications indicate that if the same distribution of u_i is used across functional specifications, it is likely to have no bearing on the ranking of farms' TE from those specifications.

4.5.1.5 Distribution of ui and Choice of Functional Form

The extent to which the adopted distribution of u_i may affect the estimation of alternative functional specifications was further examined in the test for the choice between the Cobb-Douglas and translog specifications. Given that the Cobb-Douglas is nested in the translog, the null hypothesis that the interaction terms of the translog are all zero was tested, implying that there is no significant difference between the translog and Cobb-Douglas specifications. The likelihood ratio (LR) test was conducted using values of the loglikelihood functions for the translog and Cobb-Douglas specifications for each of the exponential, half-normal and truncated-normal distributions of u_i.

The test results are presented in Table 4.18. Across all three distributions, the null hypothesis was rejected, implying that the translog is more likely to be the correct specification for the population from which the sample was drawn than is the Cobb-Douglas specification. In addition, the values of the corresponding likelihood functions across the distributions are very close, which is further indication that the adopting of alternative distributions of u_i is likely to have no effect on the estimation results across functional specifications. This is what is observed in both Figures 4.2 and 4.3, which show for the same distribution assumption across functional specifications similar distributions of efficiency estimates.

4.5.2 Cost Frontier Results

In this sub-section, the results from cost frontier estimations are presented and discussed in the same manner as in the previous section regarding estimation of production frontiers. Three Cobb-Douglas cost frontiers were estimated, alternatively assuming the exponential, half-normal, and truncated-normal distributions u_i. However, for the translog specification, only frontiers for the half-normal and truncated-normal distributions were successfully estimated, as noted before. This sub-section therefore reports and discusses coefficient estimates of parameters for those frontiers (4.5.2.1), economic efficiency (EE) measures (4.5.2.2), correlation coefficients of economic efficiency estimates (4.5.2.4), and the LR test for the equality of Cobb-Douglas and translog specifications for each of the three distributions (4.5.2.5).

4.5.2.1 Estimates of the Cost Frontier Parameters

The coefficient estimates for the parameters of the cost frontiers are presented in Tables 4.19 and 4.20 for the Cobb-Douglas and translog specifications, respectively. In the Cobb-Douglas frontiers (Table 4.19), the coefficient estimates on the variables are positive and statistically significant, except for the coefficient on the price of hay and forage (i.e., PHF), which is negative and statistically insignificant. This is similar to the case for the coefficient estimate on hay and forage (HF) in the estimation of the production function frontiers. In addition, the coefficients on similar variables across alternative distributions of u_i do not vary significantly in magnitude, an indication that the estimated frontiers are likely to be the same.

Table 4.18 *LR Test for Equality of Cobb-Douglas and Translog Production Frontiers*

	EXPONENTIAL	HALF-NORMAL	TRUNCATED-NORMAL
H_0: All interaction terms of the translog frontier are equal to zero, simultaneously.			
LLF_u[a]	542.62	545.00	553.53
LLF_R[a]	496.45	498.99	505.29
LR[b]	92.36	92.02	96.48
Critical Value (5% level)	12.59	12.59	12.59
Decision	Reject H_0	Reject H_0	Reject H_0

[a] LLF_U and LLF_R are the log likelihood function values of the unrestricted and the restricted functions, respectively.

[b] LR is the computed Likelihood Ratio value

The estimation of translog frontiers was beset with problems from the start, when the parameters of the loglikelihood function for the exponential distribution could not be obtained. In addition, most of the coefficient estimates for the translog frontiers (Table 4.19) are not statistically significant across the alternative distributions of u_i, except mainly for the coefficients on the price of hay and forage (PHF) and some of its interaction terms. This is the reverse of the results of the estimations for the Cobb-Douglas cost frontiers. The statistical significance of the price of hay and forage and its interaction terms, combined with the statistical insignificance of other variables, may signify a problem that could be linked to hay and forage (HF), which showed a similar trend in the estimation of production frontiers.

4.5.2.2 Economic Efficiency (EE) Measures

Since, as noted previously, the results for the exponential distribution could not be obtained for the translog frontier estimates, the comparisons with respect to economic efficiency are made primarily between the half-normal and truncated normal distributions. The average measures (Table 4.21) show that mean EE for the truncated and half-normal distributions is very high and almost of equal magnitude (0.94-0.95) in both the Cobb-Douglas and translog frontiers. The corresponding median EE values for the two distributions are almost identical to mean EE, except in Cobb-Douglas frontier where the half-normal distribution is higher by a percentage point (0.94 versus 0.95). For the exponential distribution, the mean and median EE are lower, relative to the other distributions (0.82 and 0.84, respectively). In general, the results indicate that about half of the farms have lower per unit costs than the "average" farm, as was the case for the production frontiers.

Table 4.19 *Coefficient Estimates for the Parameters of Cobb-Douglas Cost Frontiers, by Alternative Distribution Assumption for u_i*

VARIABLE	EXPONENTIAL		HALF-NORMAL		TRUNCATED-NORMAL	
	COEFF.	T-RATIO	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	2.837*	19.378	2.700*	17.238	2.734*	20.244
Output (Q)	0.494*	13.329	0.513*	12.942	0.507*	14.855
Price[a] of						
Grains and Concentrates (PGC)	0.150*	6.779	0.142*	5.917	0.161*	7.796
Hay and Forage (PHF)	-0.011	-0.798	-0.010	-0.652	-0.014	-1.065
Capital (PK)	0.322*	11.408	0.327*	10.869	0.328*	13.176
Other Inputs (POI)	0.412*	11.923	0.410*	11.206	0.404*	13.178
Year (YR)	0.494*	13.329	0.513*	12.942	0.507*	14.855
θ	0.012*	6.251	0.012*	5.642	0.012*	6.483
σ_v	6.479*	17.827	-	-	-	-
λ	0.129*	23.755	-	-	-	-
σ	-	-	2.379*	17.232	6.132*	3.838
μ/σ_u	-	-	0.298*	37.328	0.624*	3.701

[a] Prices are relative to the price of labor

* Coefficients are significant at 5% percent.

Table 4.20. *Coefficient Estimates for the Parameters of Translog Cost Frontiers, by Alternative Distribution Assumption for u_i*

	HALF-NORMAL		TRUNCATED NORMAL	
VARIABLE	COEFF.	T-RATIO	COEFF.	COEFF.
Constant	5.851*	3.127	5.804*	3.259
Output (Q)	-0.991	-1.084	-1.022	-1.179
Price[a] of				
Grains and Concentrates (PGC)	0.818	1.581	0.712	1.355
Hay and Forage (PHF)	-0.815*	-2.236	-0.828*	-2.314
Capital (PK)	-0.199	-0.237	-0.092	-0.109
Other Inputs (POI)	1.235	1.230	1.305	1.274
Year (YR)	0.057	1.236	0.060	1.320
PGC.PGC	0.028	0.207	-0.079	-0.588
PGC.PHF	-0.100*	-1.938	-0.082	-1.585
PGC.PK	0.027	0.220	0.060	0.470
PGC.POI	0.056	0.343	0.111	0.675
PGC.Q	-0.137	-1.048	-0.112	-0.846
PGC.YR	-0.009	-1.186	-0.006	-0.788
PHF.PHF	0.106*	2.475	0.118*	2.852
PHF.PK	-0.012	-0.159	0.011	0.149
PHF.POI	0.003	0.036	-0.053	-0.568
PHF.Q	0.193*	2.086	0.192*	2.128
PHF.YR	0.000	0.035	-0.001	-0.270
PK.PK	-0.430	-1.812	-0.586*	-2.426
PK.POI	0.449	1.812	0.543*	2.158
PK.Q	0.193	0.914	0.179	0.845
PK.YR	-0.003	-0.292	-0.002	-0.145
POI.POI	-0.547	-1.593	-0.630	-1.805
POI.Q	-0.304	-1.202	-0.328	-1.275
POI.YR	0.015	1.048	0.013	0.880
Q.Q	0.358	1.586	0.374	1.754
Q.YR	-0.013	-1.119	-0.014	-1.247
YR.YR	0.001	1.393	0.001	1.314
θ	-	-	-	-
σ_v	-	-	-	-
λ	6.568	6.568	3.906	7.332
σ	0.125	0.125	0.412	6.922
μ/σ_u	-	-	-1.038	-1.445

[a] Prices are relative to the price of labour

* Coefficients are significant at 5% percent.

Although the half-normal and truncated-normal distributions have similar mean EE values, the distributions of EE estimates (Figure 4.3) are not identical. Specifically, the half-normal distribution results in twice the number of farms with EE greater than 0.95 as for the truncated-normal distribution. However, with the truncated-normal distribution, all of the firms have EE in the range of 0.93 to 0.99, whereas for the half-normal EE values are between 0.72 and 0.95. This pattern is similar to the TE distribution results, discussed earlier. This is reflected in the variance between EE estimates, which is smaller for the truncated normal distribution (0.0002) than for the half-normal distribution (0.001).

The variance for estimates from the exponential distribution is even larger (0.09). Thus, the distributions of EE show that the estimates of the exponential distribution are more widely spread (mainly between 0.70 and 0.97) relative to EE distributions for the half-normal and truncated-normal distributions, which have values that are mainly 0.90 in both specifications.

The pattern that was observed in production frontiers, whereby the half-normal ranked highest, is the same with the cost frontiers. However, the results for the exponential distribution should be interpreted with caution. Since the translog cost frontiers could not be estimated with the exponential distribution, it is likely that even the estimates from the Cobb-Douglas frontier for the distribution may be problematic as well, albeit to a smaller degree.

4.5.2.3 Correlation Coefficients of EE Estimates

The pair-wise rank correlation coefficients of EE estimates were used to further examine the effect of the distribution of u_i on EE estimates. Table 4.21 reports the computed rank correlation coefficients, which are summarized in three groups, similar to TE correlation coefficients. All of the reported coefficients were found to be statistically significant at 1% level. The first group (upper left) contains pair-wise correlation coefficients of EE estimates between the three distributions for the Cobb-Douglas specification.

Table 4.21 *Economic Efficiency (EE) Measures for Cobb-Douglas and Translog Production Frontiers*

	COBB-DOUGLAS			TRANSLOG	
	EXPONENTIAL	HALF-NORMAL	TRUNCATED-NORMAL	HALF-NORMAL	TRUNCATED-NORMAL
Mean EE	0.82	0.94	0.94	0.95	0.94
Median EE	0.84	0.95	0.94	0.95	0.94
Max. Value	0.97	0.98	0.99	0.98	0.99
Min. Value	0.47	0.73	0.93	0.72	0.93
Variance	0.09	0.001	0.0002	0.001	0.0002

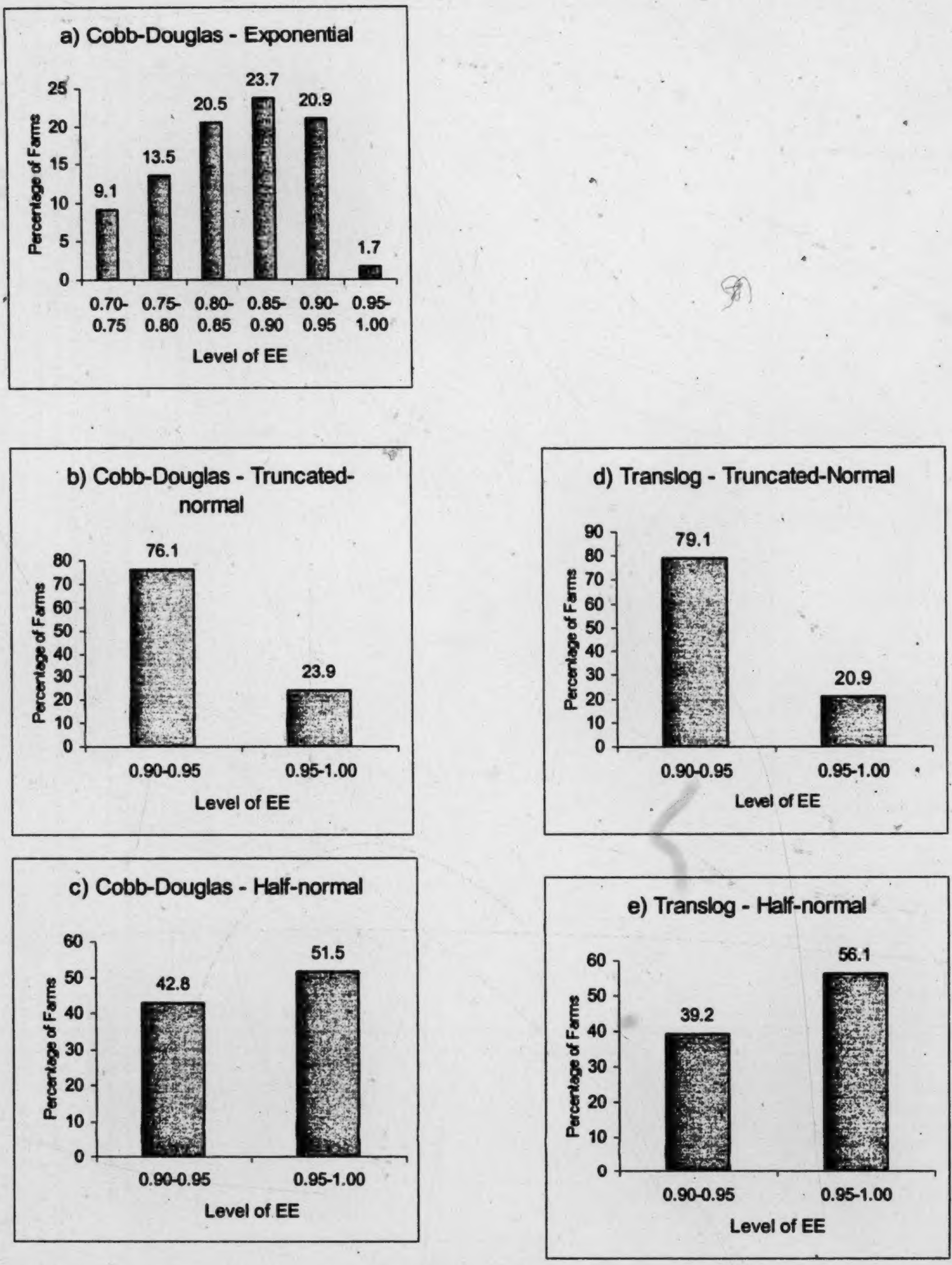

Figure 4.5 Distribution of Economic Efficiency (EE) in Terms of Percentage of Farms, by Functional Form and Alternative Distributional Assumption for u_i

The second group (upper right) represents correlation coefficients of estimates between the Cobb-Douglas and translog specifications for each particular distribution. The third group (lower right) represents correlation coefficients of EE estimates between the two distributions for the translog specification. The magnitudes of the correlations vary by functional specification. For the Cobb-Douglas estimates, the correlation coefficient is positive and high between the EE estimates for the half-normal and truncated-normal distributions (0.946), whereas the correlation coefficients between the exponential and other distributions are relatively low (0.386 and 0.576), but positive. In the case of the translog frontier (third group), the correlation coefficient for the half-normal and truncated-normal distributions is also high and positive (0.909), as is the case with the Cobb-Douglas formulation. Based on the Cobb-Douglas frontiers, the ranking of EE estimates by farms for the half-normal and truncated-normal distributions are closer, while those for the exponential distribution are different from the other two.

The available translog frontier results indicate that the ranking of farms between the half-normal and truncated-normal distribution reflects the same pattern as the Cobb-Douglas frontiers. These findings complement the results for both the mean EE and the respective variances, which indicated a similarity in magnitude between truncated-normal and half-normal distributions, and a difference in these results from those for the exponential distribution. The correlation coefficients of EE estimates for the same distribution across functional specifications (second group) are not only high but also very close in magnitude (0.942 and 0.948). As was the case with production frontiers, the high correlation coefficients by distributions across the two functional specifications is further indication that the chosen distribution of u_i may not affect the EE estimates from various specifications.

Table 4.22 *Spearman's Rank Correlation Coefficients of Economic Efficiency (EE) Estimates from Cobb-Douglas and Translog Production Frontiers for Alternative Distribution Assumptions*

		COBB-DOUGLAS			TRANSLOG	
		EXPONENTIAL	TRUNCATED-NORMAL	HALF-NORMAL	TRUNCATED-NORMAL	HALF-NORMAL
COBB-DOUGLAS	EXPONENTIAL	1.000	0.386	0.576	0.354	0.357
	TRUNCATED-NORMAL		1.000	0.946	0.942	0.901
	HALF-NORMAL			1.000	0.542	0.948
TRANSLOG	TRUNCATED-NORMAL				1.000	0.909
	HALF-NORMAL					1.000

4.5.2.4 Distribution of u_i and Choice of Functional Form

In examining the extent to which the adopted distribution of u_i may affect the estimation of alternative functional specifications, the test for the choice between the Cobb-Douglas and translog specifications was conducted for all distributions. Given that the Cobb-Douglas is nested within the translog formulation, the null hypothesis that the interaction terms of the translog are all zero was tested; that is, testing whether or not the Cobb-Douglas is an appropriate specification. The likelihood ratio (LR) test was conducted using values of the loglikelihood functions for the translog and Cobb-Douglas specifications for each of the half-normal and truncated-normal distributions of u_i. The test results are presented in Table 4.23.

As was the case with the production function frontiers, the null hypothesis was rejected for both distributions. This implies that the translog is more likely to provide the correct specification of the parameters of the population from which the sample was drawn, since the inclusion of the interaction terms is statistically significant. Since the results are similar across distribution assumptions, it may be implied that the adopting of alternative distributions of u_i is likely to have no effect on the estimation results across functional specifications with regard to a particular distribution chosen.

Table 4.23 *LR Test for Equality of Cobb-Douglas and Translog Cost Frontiers*

	HALF-NORMAL	TRUNCATED-NORMAL
H_0: All interaction terms of the translog frontier are equal to zero, simultaneously.		
LLF_U[a]	264.88	246.52
LLF_R[a]	246.52	223.02
LR[b]	49.26	47.00
Critical Value (5% level)	12.59	12.59
Decision	Reject H_0	Reject H_0

[a] LLF_U and LLF_R are the log likelihood function values of the unrestricted and the restricted functions, respectively.

[b] LR is the computed Likelihood Ratio value.

4.5.3 Discussion of Results on the Effect of u_i on Efficiency Measures

The foregoing analysis in sections 4.5.1 and 4.5.2 has attempted to examine the sensitivity of efficiency measures to the distribution of the inefficiency error term used in the estimations, and to answer whether or not distributional assumptions matter in these estimations. If distribution assumptions are important, then to what extent are efficiency measures from various distributions different? If the difference is sufficiently great that it may not be appropriate to compare levels of measures based on different distributions, then are the resulting measures at least ranked in the same pattern for firms in the sample?

In examining this sensitivity of TE measures to distribution assumptions, this study has drawn from previous literature. One approach has been to compare the results of the exponential and half-normal distributions (e.g., Rossi & Canay, 2001). This is based on the argument that simpler distributions ought to be adopted since, if Ritter and Simar's (1997) assertion is true, distributions do not make a difference in the results. Second, comparison was sought between simpler distributions and more flexible distributions (i.e., the truncated-normal). This comparison is an issue because distributions are likely to be data-specific (Mbaga et al., 2000) and there may well be a similarity between the truncated-normal and half-normal distributions, since the latter is nested in the former. Third, comparison was made with regard to particular distributions across functional specifications, in order to provide evidence as to whether the results from adopting a particular distribution are consistent across functional forms.

The comparison between the exponential and half-normal distributions was made in two ways. The first was to compare the levels of estimates between the two distributions, based on the average measures of efficiency; namely, the mean and median. The second comparison was to examine the correlation coefficients between the measures as well as the variance in order to gauge whether the ranking for the two sets of estimates are different. The results were consistent for the production and cost frontiers: the half-normal distribution resulted in higher average TE and EE estimates than did the truncated-normal distribution. For both TE and EE, the rank correlation coefficients of efficiency measures between the two distributions were low, but positive, as opposed to high and positive correlations between the half-normal and truncated-normal distributions.

In the reviewed studies, evidence of correction is mixed with a tendency for correlation coefficients between efficiency measures for pairs of distributions to be higher rather than lower. The low correlation coefficient estimates between the exponential distribution and other distributions and high correlation between the half-normal and truncated normal distributions found in this study may lend further evidence to the conclusion by Mbaga et al. (2000) that the resulting estimates are specific to the data used.

The results of the comparisons among the estimates of the three distributions touch on the issues raised earlier. First, the closeness of the results between the truncated-normal and the half-normal distributions may indicate that the choice of the distribution (i.e., simple versus flexible) does not matter; on the other hand, the low correlation between the exponential and other distribution tend to indicate the opposite; that is, the choice of the distribution does matter. The latter is further indicated by the distributions of efficiency measures that show differences, especially for percentage of farms above 0.95 and lower than 0.90, whereby the half-normal distribution resulted in a higher percentage of farms in this range than both the exponential and truncated normal distributions and the truncated normal more than the exponential distribution.

Lastly, the measures for a given distribution are ranked very closely across functional specifications, in addition to being very close in magnitude. Thus, comparison of results across specifications should be with respect to a particular distribution assumption rather than across distributions. In testing the appropriateness of the Cobb-Douglas specification, given the specification of the translog frontier, the hypothesis was rejected in all three distributions, which indicates that the Cobb-Douglas does not fit the data as well as the translog specification.

However, most of the coefficients for the parameters of the translog frontiers were statistically insignificant, in addition to some of the elasticities of output being negative or too large relative to those of the Cobb-Douglas frontiers.

This examination points to two discernible patterns. First, the half-normal and truncated normal distribution results are very similar. Although the half normal is a relatively simpler distribution, it is nested in the truncated normal distribution, which may explain the closeness of the results. The second pattern is in regard to closeness of estimates across specifications for a given distribution. Other hypotheses have not been found to be true. The estimates of the exponential distribution were not found in general to be closer to the half-normal, neither were they to the truncated normal; moreover, the ranking of farms by efficiency measures (TE and EE) differed across distributions. Therefore, the conjecture is that the pattern discerned from efficiency estimates from various distributions may be specific to the data used, indicating that the assumed distribution matters in terms of the results.

Chapter 5. Summary and Conclusions

The main objective of this study was to investigate the efficiency -- technical as well as economic – for milk production in Alberta. Because dairy farmers in North America are facing continuing changes in technological, structural and economic environment, which are likely to continue into the future, Canadian dairy producers will be exposed to more competition. Hence, emphasis on improving on efficiency and management practices of dairy undertakings is important for Alberta dairy farmers. In order to do that, however, they need to have an indication of how efficient their operations are now and what factors influence this efficiency.

5.1 Summary of Model

Technical and economic efficiency were examined using stochastic production and cost frontiers, respectively. The data for a sample of Alberta milk producers, spanning the periods 1980 –1996 and 1986 – 1996, were fitted to the Battese-Coelli (1995) model using econometric techniques to generate estimates of the frontiers and efficiency measures. While technical efficiency was estimated as part of the total error term of the stochastic production frontier, economic efficiency was estimated as part of the total error term of the cost frontier. Production and cost frontiers were, respectively, gauged on the output and cost of production of best performing farms.

Based on this maximum efficiency for a sample of Alberta dairy producers, the mean technical (TE) and economic (EE) efficiency values for the whole sample (1980-96) and the sub-sample (1986-96) were computed as simple averages of individual farm efficiency. Efficiency was further examined in terms of how it varied over the sample period. Furthermore, the estimated production frontiers were used to compute the elasticities of inputs and returns to scale (RTS). In addition, since the distribution of the inefficiency error term was not known a priori, the study examined the sensitivity of the estimates to the distribution assumption of the inefficiency error term, based on the Jondrow et al. (1982) model. In this regard, the models were re-estimated using LIMDEP (Greene, 2000).

An attempt was made to decompose economic efficiency into its components of technical and allocative efficiency using the Kopp-Diewert (1982) approach. This approach invokes Shephard's Lemma to determine a vector of prices that would lead to optimal cost at the point along the isoquant that is technically but not allocatively efficient (point B). Allocative efficiency (AE) is then computed as the ratio of the minimum cost of production (as calculated from the estimated cost frontier function) to the cost of production at point B. Technical efficiency (TE) is given by the ratio of the cost at point B to the actual cost incurred. This decomposition, however, did not give satisfactory results, for which reasons they have not been reported. Instead, allocative efficiency was calculated for the self-dual functional form (i.e., the Cobb-Douglas) from the estimates of technical and economic efficiency.

The above analyses were coupled with determination of potential sources of technical and economic inefficiency for Alberta dairy farms, by empirically examining and elaborating on the influence of factors in the models explaining either the technical inefficiency or the economic inefficiency of Alberta milk producers. The estimation of these efficiency models involved regressing, on the estimated inefficiency (TE or EE models), a set of variables hypothesized to explain the levels of inefficiency. These models were estimated econometrically simultaneously with the corresponding frontiers. Finally, several Loglikelihood ratio (LR) tests were conducted to analyse the impact of different methodological assumptions about stochastic frontiers.

5.2 Summary of Empirical Results

The results for the production and cost frontier estimates were hinged on the related functional formulations used. In terms of statistical significance and economic interpretation of the estimated parameters, the performance of the Cobb-Douglas and the simplified translog formulations was good, whereas the performance of the conventional translog formulation was not. However, the results of statistical tests as to whether the Cobb-Douglas and the simplified translog formulations were invariant from the conventional translog formulation implied that neither the Cobb-Douglas nor the simplified translog could be chosen over the conventional translog formulation.

The choice as to which results should be adopted between those of the Cobb-Douglas and simplified translog formulations on the one hand and the conventional translog formulation on the other was a dilemma between economic sense in the interpretation of results and flexibility of the formulations. The conventional translog formulation is normally suspected to suffer from some estimation problems. These problems are likely to have affected its parameter estimates such that they were at variance with expected results. An examination as to whether the variables in the models were linearly related indicated that they were not. The 'goodness' of the results from the Cobb-Douglas and simplified translog formulations were coupled with the inflexibility of these formulations. Thus at the expense of inflexibility, the results of the Cobb-Douglas and simplified translog formulations that rendered themselves to sensible economic interpretation were adopted.

The mean technical efficiency (mean TE) indicates that Alberta farmers are on average 91 percent technically efficient, based on the whole sample period, and 87 percent technically efficient when the sub-sample period is considered. This implies that Alberta farmers could still use the same amount of inputs and improve milk output, on average, by 9–13 percent through enhancement of the physical productivity of inputs used in production. The mean economic efficiency (Mean EE) is approximately 84 percent, which implies that the Alberta dairy farmers could reduce the cost of milk production by 16 percent by both utilizing the inputs efficiently and allocating them optimally according to their relative prices. The median of economic efficiency (EE) measures indicate that more than 50 percent of the farms in both sample periods are performing better than the average farm. The computations of allocative efficiency indicated that the mean allocative efficiency (Mean AE) is approximately 92 percent, which implies that the average farm would reduce its cost by about 8 percent if it were to reallocate the used inputs in accordance with their relative prices.

In analysing potential sources of technical or economic efficiency for Alberta dairy farms, a number of factors were identified. Herd size was shown to exert a negative and significant influence on economic efficiency, and insignificant influence on technical efficiency, which indicates that small herds tend to show higher levels of efficiency than large ones. Though large farms may imply high levels of economies of scale, this may come at the expense of reduced intensity in animal care, which may explain why the influence of herd size on technical efficiency is not statistically significant, since reduced intensity in animal care may lead to a decrease in cows' productivity.

Capital intensity was found to be more important in enhancing output than labour intensity. Though capital is more expensive than labour, the results tend to suggest an incentive exists to utilize more of it relative to labour because of its relatively higher contribution to productivity.

Higher breeding and veterinary services were found to be associated with high levels of technical efficiency, and vice versa for economic efficiency. The implication is that though breeding and veterinary services may not result in reduced cost per cow, they tend to enhance output. Lastly, the increase in the ratio of grains and concentrates to hay and forage was found to enhance technical efficiency, though it was not significant with respect to economic efficiency.

Time was found to have no bearing on either technical or economic efficiency, which implies that in the period examined, changes in the levels of technical and economic efficiency have not been significant. The stochastic model was found to be a correct representation of both the production and cost frontiers relative to either "average functions or deterministic frontier function". Lastly, different distributions of the inefficiency error term were found to be inconsistent in terms of resulting average measures and the ranking of individual technical and economic efficiency measures, indicating that the appropriate choice of the distribution is dependent on the data used.

5.3 Conclusions

This study set as its principal objective to determine technical and economic efficiency for milk production in Alberta. Using a farm-level panel data set of Alberta dairy farms, empirical results suggest that Alberta dairy farms are, on average, highly efficient relative to the best farms in the industry. This suggests that the farms are relatively homogeneous in their efficiency of production. These results are consistent with the findings of previous studies that have analysed the Alberta dairy industry (e.g. Richards & Jeffrey, 1996, 1998, 2000).

In addition to the estimation of efficiency, analysis of potential sources of technical or economic efficiency for Alberta dairy farms suggests a number of additional implications. First, Alberta dairy farmers have the potential of improving on their performance, on average, both in terms of utilization of inputs and reduced costs – even without resorting to new technologies. To this effect, Alberta farmers may consider spending more on breeding and veterinary services, exploiting opportunities availed by capital and utilising fully the existing scale of operation. Secondly, marginal gains in reduction of cost of producing milk may come through reallocation of inputs.

Lastly, as Alberta farmers are encouraged to embark on herd expansion, as a strategy for enhancing competition (Western Producer, 1999), the findings of this study with regard to a positive relationship between herd size and economic efficiency need to be qualified. Richards and Jeffrey (2000) suggest that the maximum economic efficiency herd size for Alberta approximates 70 cows, which refers to the economies of size and not to the herd size that reflects high levels of economic efficiency. Small farms are likely to have lower capital-labour ratio than large farms. Also, the results obtained are conditional to a given (fixed) level of all other explanatory variables that lower than the present (approximate) average herd size in the province. In addition, with the findings in this study that the average farmer is operating in the economically rational zone, herd expansion may have to go hand in hand with additional capital investment. This goes beyond the mere raising of efficiency levels to adopting new techniques. Though this may require big investment, it may be timely, especially because the potential gains from only efficiency improvements are now limited, given that the farms have already reached high levels of efficiency and are highly homogeneous in their operations.

5.4 Limitations and Directions for Further Research

In conducting this study, several limitations were faced. The main problems related to the quality of some data, or lack of it. For example, because the specificity of individual farms could not be ascertained, analyses of this study were confined to the "average farm"; also, some data on labour were incomplete and had to be generated by averaging from the available data. Moreover, some information for family labour on wages and hours worked might have been best estimates by the respondents, for lack of record. Data shortcomings may be the cause of some of the results that did not augur with economic interpretations, for which reasons the results were not reported. Good results are primarily predicated on the goodness of data (in terms of both quantity and quality) as well as on the closeness of the theoretical model to the real world it attempts to capture, for example, the chosen functional form. To the extent that the data were deficient in some respects, these shortcomings are likely to have compromised some of the results. In addition, the data were collected from producers operating in a regulated environment of supply management; thus, their economic decisions are likely to have been constrained within this policy regime and may change if the environment were to change.

Another limitation is related to the scope of the study. Like many related studies on efficiency in North America, the analyses focused on only one region. This focus makes comparisons with other studies very difficult in view of the differences in approaches, methods, and methodological assumptions, even if data limitations were to be assumed away and basic assumptions about economic theory were to be invoked.

Future studies within provinces should focus on estimating efficiency using different approaches and methods. A few studies that have attempted this analysis have indicated that the estimated efficiency levels are not the same across approaches or even for different methods within a particular approach, even though the ranking of firms' performance is maintained. If this were to be further evidenced, the magnitudes of efficiency levels would be a relevant gauge of performance only when the methods are the same. For parametric estimation, a two-step procedure is recommended. First, frontiers should be estimated using alternative distribution assumptions for the inefficiency error term. Second, tests for the best functional formulation and further economic analyses should follow, based on the distribution of inefficiency that is indicated as being specific to the data used.

Secondly, because the main objective of these studies is to enable the producers to assess their competitiveness, it is not enough to gauge their performance against the best in their own industry. Studies need to be broadened to include several provinces. Researchers in different provinces could pool resources and data to carry out such studies. However, because Canada has not yet adopted some of the technologies used in the US, which are affecting the yield per cow, it may not be advisable to carry out comparative studies that rely on aggregated output data involving US states. Good comparative studies of efficiency would require the output be disaggregated and related to the underlying technology. Alternatively, perhaps it is time we considered estimating "health-sensitive" production and cost frontier models for the dairy undertakings in North America.

References

Afriat, S. H. (1972). "Efficiency Estimation of Production Functions". *International Economics Review*, *13*, 569-598.

Agriculture and Agri-Food Canada (1996). "The Milk and Dairy Products Industry". In R. M. A. Loyns, K. Meilke, & R. D. Knutson (Eds.), *Understanding Canada/United States Dairy Disputes*, Proceedings of the Second Canada/U.S. Agricultural and Food Policy Systems Information Workshop, University of Guelph

Ahmad, M., & Bravo-Ureta B. E. (1995). An Econometric Decomposition of Dairy Output Growth. *American Journal of Agricultural Economics*, *77*, 914-921.

Ahmad, M., & Bravo-Ureta B. E. (1996). Technical Efficiency Measures for Dairy Farms Using Pannel Data: A Comparison of Alternative Model Specifications. *Journal of Productivity Analysis*, *7*, 399-415.

Aigner, D. J., & Chu S. F. (1968). On Estimating the Industry Production function. *American Economic Review*, *58*, 826-839.

Aigner, D. J., Lovell, C. A. K., & Schmidt, P. (1977). Formulation and Estimation of Stochastic Frontier Function Models. *Journal of Econometrics*, *6*, 21-37.

Alberta Agriculture. Various years. *Economics of Milk Production*. Production Economics Branch, Edmonton, Alberta, Canada. **http://www.albertamilk.com/journey/cows_barns/index.asp**

Ali, A. I. & Seiford, L. M. (1993). The Mathematical Programming Approach to Efficiency Analysis. In H. O. Fried, C. A. K. Lovell, & S.S. Schmidt (Eds.). *The Measurement of Productive Efficiency* (pp.120-159). New York: Oxford University Press.

Amara, N., Traore, N., Landry, R., & Romain R. (1999). Technical Efficiency of Farmers Attitude Toward Technological Innovation: The Case of the Potato Farmers in Quebec. *Canadian Journal of Agricultural Economics*, *47(1)*, 31-49.

Ball, V.E. (1985). Output, Input and Productivity Measurement in U.S. Agriculture, 1948 – 1979. *American Journal of agricultural Economics*, *67*, 475-486.

Barichello, R. R., Lambert, R., Lambert, T. J., Richards, Romain, R.F., & Stennes B. K. (1996). Cost Competitiveness in the Canadian and US Dairy Industires. In A. Schmitz, G. Coffin, & K.A. Rosaasen (Eds.), *Regulation and Protectionism Under GATT: Case Studies in North American Agriculture* (pp.96-117). Boulder:Westview.

Battese, G.E. (1992). Frontier Production Function and Technical Efficiency: A Survey of Empirical Applications in Agricultural Economics. *Agricultural Economics*, 7,185-208.

Battese, G. E., & Broca, S. S. (1996). Functional Forms of Stochastic Frontier Production Functions and Models for Technical inefficiency Effects: A comparative Study For Wheat Farmers in Pakistan, *CEPA Working Papers, No.4/96.* Department of Econometrics, University of New England.

Battese, G. E., & Coelli, T. J. (1988). Prediction of Firm-Level Technical Efficiencies with a Generalised Frontier Production Function and Panel Data. *Journal of Econometrics 38*, 387-399.

Battesse, G. E., & Coelli, T. J. (1992). Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India. *Journal of Productivity Analysis 3*, 153-169.

Battese, G. E., & Coelli T. J. (1995). A Model for Technical Inefficiency Effects in a Stochastic Frontier production Function for Panel Data. *Empirical Economics*, *20*, 325-332.

Battese, G. E., & Corra, G.S. (1977). Estimation of a Production Frontier Model: With Application to the Pastoral Zone of Eastern Australia, *Australian Journal of Agricultural Economics, 21*, 169-179.

Bauer, P.W. (1990). Recent Developments in the Econometric Estimation of Frontiers. *Journal of Econometrics*, *46*, 39-56.

Bauer, P.W. (1985). *An Analysis of Multi-product Technology and Efficiency Using the Joint Cost Function and Panel Data an Application to the U.S. Airline Industry*. Unpublished Doctoral Dissertation, University of Carolina.

Bravo-Ureta, B. E. (1986). Technical Efficiency measures for Dairy farms Based on a Probabilistic Frontier Function Model. *Canadian Journal of Agricultural Economics, 34*, 399-415.

Bravo-Ureta, B. E, & Evenson, R. E. (1994). Efficiency in Agricultural Production: The Case of Peasant Farmers in Eastern Paraguay. *Agricultural Economics*, *10,* 27- 37.

Bravo-Ureta, B. E., & Pinheiro, A. E. (1993). Efficiency Analysis of Developing Country Agriculture: A Review of the Frontier Function Literature. *Agricultural and Resource Economics Review*, *22*, 88-101.

Bravo-Ureta, B. E., & Rieger, L. (1990). Alternative Production Frontier Methodologies and Dairy Farm Efficiency. *Journal of Agricultural Economics*, *41*, 215-226

Bravo-Ureta, B. E. & Rieger, L. (1991). Dairy Farm Efficiency Measurement Using Stochastic frontiers and Neoclassical Duality. *American Journal of Agricultural Economics*, *73*, 421-428.

Cameron, B., & Gould, R. (1998). *Dairy Farming in Alberta*. Dairy Extension Advisory Group.

Canadian Dairy Commission (2002). *Pricing and Subsidies.* **http://www.cdc.ca/cdc/main_e.asp?catid=283**

Canadian Dairy Commission (2000). *Supply Management in Canada*. **http://www.cdc.ca/msupply.html**

Canadian Dairy Commission (2000). *Canada's National Dairy Policy Framework.* **http://www.cdc.ca/dpolicy.html**

Carlson, G.A., Zilberman, D., & Miranowski, J. A. (1993). *Agricultural and Environmental Resource Economics.* New York: Oxford University Press.

CEER (2000). *The Relative Performance of Electricity Distribution firms in South America*. Mimeo, Centro De Estudios Economicos de la Regulacion (CEER), Univesrsidad Argentina de la Empresa.

Chambers, R. G. (1988). *Applied production analysis: A dual approach*. New York: Cambridge University Press.

Charnes, A.W., Cooper, W., & Rhodes, E. (1978). Measuring the Efficiency of Decision Making Units. *European Journal of Operations Research*, 2, 429-444.

Cloutier, L. M., & Rowley, R. (1993). Relative Technical Efficiency: Data Envelope Analysis and Quebec Dairy Farms. *Canadian Journal of Agricultural Economics*, *41*,169-176.

Coelli, T. J. (1994). *A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation*. Mimeo, Department of Econometrics, University of New England, Amidale.

Coelli, T. J. (1995a). Recent Developments in Frontier Modelling and Efficiency Measurement. *Australian Journal of Agricultural Economics*, *39*, 219-245.

Coelli, T. J. (1995b). Estimators and Hypothesis Tests for a Stochastic Frontier Function: A Monte Carlo Analysis. *Journal of Productivity Analysis*, *6*, 247-268.

Coelli, T.J. (1996). *A Guide to FRONTIER Version 4.1: A computer Program for Frontier Production Function Estimation. CEPA Working Paper 96/07*. Department of Econometrics, University of New England, Armidale

Coelli, T.J., & Battese, P. (1996). Identification of Factors which Influence and Technical Inefficiency of Indian Farmers. *Australian Journal of Agricultural Economics*, *40*,103-28.

Coelli, T., Prasada Rao, D.S., & Battese, G.E. (1998). *An Introduction to Efficiency and Productivity Analysis*. Boston: Kluwer Academic Publishers

Cordon, S. (2002). *Back to Business for Canadian Dairy Farmers after WTO win in lengthy dispute*. **http:/ca.news.yahoo.com/011203/6/f65a.html**

Cornwell, C., P. Schmidt, and R. C. Sickles (1990). 'Production Frontiers with Cross sectional and Time Series Variation in Efficiency Levels', *Journal of Econometrics, 46*,185-200.

Debreu, E. (1951). 'The Coefficient of Resource Utilization. *Econometrica, 19*, 273-292.

Durr, J. W., Monardes, H. G., Cue, R. I., & Philpot, J. C. (1997). Culling in Quebec Holstein Herds. 1. Study of Phenotypic Trends in Herd Life.' *Canadian Journal of Animal Science, 77*, 593-600.

Estache, A., & Rossi, M. (1999). *How Different is the Efficiency of Public and Private Water Companies in Asia?* Policy Research Paper No. 2152. Washington: The World Bank.

Fan, S. (1991). Effects of Technological Change and Institutional Reform on Production Growth in Chinese Agriculture. *American Journal of Agricultural Economics, 73*, 216-275

Fare, R., & Lovell, C. A. K. (1978). Measuring the Technical Efficiency of Production', *Journal of Economic Theory, 19*, 150-162.

Fare, R., & Lovell, C. A. K. (1985). *The Measurement of Productive Efficiency*. Dordrecht Kluwer-Nijhoff.

Farrell, M. J. (1957). The Measurement of Productive Efficiency. *Journal Royal Statistical Society, ACXX, Part 3*, 253-290.

Farrell, M. J. & Fieldhouse M. (1962). Estimation Efficient Production under Increasing Returns to Scale. *Journal of the Royal Statistical Society A125 part* 2, 252-267.

Foley, R.C., Bath, D.L., Dicknson, F. N., & Tucker H. A. (1972). *Dairy Cattle: Principles, Practices, Problems, Profits*. Philadelphia: Lea & Febiger

Forsund, F.R., Lovel C. A. K. & Schmidt, P. (1980). A Survey of Frontier Production Functions and of their Relationship to Efficiency Measurement. *Journal of Econometrics, 13*, 5-25,

Fox, G., Roberts, B., & Brinkman, G.L. (1992). Canadian Dairy Policy and the Returns to Federal Dairy Cattle Research. *Agricultural Economics, 6*, 267-85.

Giannakas, K., Tran, K.C., & Tzouvelekas, V. (1998). On the Choice of the Functional Form for Agricultural Efficiency Measurement. *Journal of Agricultural Productivity.*

Greene, W. H. (1990). A Gamma-distributed Stochastic Frontier Model, *Journal of Econometrics, 46*, 141-164.

Greene, W. H (1980). Maximum Likelihood Estimation of Econometric Frontier Functions. *Journal of Econometrics, 13*, 27-56.

Greene, W. H. (1993). The Econometric Approach to Efficiency Analysis. In H. O. Fried, C. A. K. Lovell, & S.S. Schmidt (Eds.), *The Measurement of Productive Efficiency* (pp. 68-119.New York: Oxford University Press.

Greene, W.H. (2000, June). *Simulated Likelihood Estimation of the Normal-Gamma Stochastic Frontier Function.* Paper presented at North American Productivity Workshop, Union College. New York: Schenectudy, 15-17.

Griffin R. C., Montgomery, J. M., & Rister, M. E. (1987). Selection of Functional Form in Production Function Analysis. *Western Journal Journal of Agricultural Economics, 12(2),* 216-227.

Grisley, W., & Mascarenhas, J. (1985). Operating Cost Efficiency on Pennsylvania Dairy Farms. *Northeastern Journal of Agricultural and Resource Economics, 14*, 88-95.

Growing Alberta (2000). *Dairy.* http://www.growingalberta.com/dec/dairy.asp

Heshmati, A., & Kumbhakar, S. C. (1995, April). Farm Heterogeneity and Technical Efficiency: Some Results from Swedish Dairy Farms. *Journal of Productivity Analysis, 5 (1),* 45-61.

Heshmati, A., Kumbhakar, S. C, & Hjalmarsson, L. (1995). Efficiency of the Swedish Pork Industry: A Farm Level Study using Rotating Panel Data 1976-1988. *European Journal of Operational Research, 8(3)*, 519-533.

Huang, C.J. & Liu, J-T. (1994). Estimation of Non-neutral Stochastic Frontier Production Function. *Journal of Productivity Analysis, 5*, 171-180.

Jaforullah, M. (1996). Technical Efficiency of Some manufacturing Industries of Bangladesh: An Application of the Stochastic Frontier Production Function Approach. *Bangladesh Development Studies, 24.*

Jafforullah, M., & Devilin, N. (1996). Technical Efficiency in New Zealand Dairy Industry: A Frontier Production Function Approach. *Journal of Policy Modelling, 18.*

Jeffrey, S. R. (1992, July). Relative Technical and Economic Efficiency for Canadian and U.S. Dairy Farms. *Working Paper No. 92-4*, Department of Agricultural Economics and Farm Management, University of Manitoba.

Jeffrey, S. R., & Richards, T. J. (1996). *Factors Influencing Costs of Milk Production in Alberta', Advances in Dairy Technology, 8.* Proceedings of the 1996 Western Canadian Dairy Seminar, Published by the Department of Agriculture, Nutritional and Food Sciences, University of Alberta.

Jondrow, J., Lovell, C. A. K., Materow, I. S., & Schmidt, P. (1982). On estimation o Technical Inefficiency in the Stochastic Frontier Production Function Model, *Journal of Econometrics*, *19*, 233-238.

Kalirajan, K. P. (1981). An Econometric Analysis of Yield Variability in Paddy Production. *Canadian Journal of Agricultural Economics*, *29*, 283-294

Kalirajan, K. P. (1991). On Measuring Economic Efficiency. *Journal of Applied Econometrics*, *5*, 75-86.

Kodde, D.A. & Palm, A.C. (1986). Wald Criteria for Jointly Testing Equality and Inequality Restrictions. *Econometrica*, *54*, 1243-1248.

Koopmans, T. C. (1951). An Analysis of Production as an Efficient Combination of Activities. In T.C. Koopmans (Ed.), *Activity Analysis of Production and Allocation*, Cowles Commission for Research in Economics, Monograph No. 13,Wiley, New York.

Kopp, R.J., & Smith, V.K. (1980). Frontier Production Function Estimates for Steam Electric Generation: A Comparative analysis. *Southern Economic Journal*, *47*, 1049-59.

Kopp, R.J., & Diewert, W. E. (1982). The Decomposition of Frontier cost Function Deviations into Measures of Technical and Allocative Efficiency. *Journal of Econometrics*, *19*, 319-331.

Kulak, K. K., Dekkers, J. C. M., McAllister, A. J., & Lee, A. J. (1997). Lifetime Profitability Measures for Dairy Cows and Their Relationships to Lifetime Performance Traits. *Canadian Journal of Animal Science*, *77*, 609-616

Kumbhakar, S. C. & Lovell, A. C. K. (2000). *Stochastic Frontier Analysis*. New York: Cambridge University Press.

Kumbhakar, S. C., Biswas, B., & Bailey, D.V. (1989). A Study of the Economic Efficiency of Utah Dairy Farmers: A System Approach. *Review of Economics and Statistics*, *71*, 595-604.

Kumbhakar, S. C. (1988). Estimation of Input-specific Technical and Allocative Efficiency Using Stochastic Frontier Functions. *Oxford Economic Papers, 40*, 535-549.

Kumbhakar, S. C. (1990). Production Frontiers, Panel Data and Time-Varying Technical Inefficiency. *Journal of Econometrics*, *46,* 201-211.

Kumbhakar, S. C., Ghosh, S., & McGuckin, J. T. (1991). A Generalized Production Frontier Approach for Estimating Determinants of Inefficiency in U.S. Dairy Farms. *Journal of Business and Economic Statistics 9(3),* 279-286

Lovell, C. A. K. (1996). Applying Efficiency Measurement Techniques to the Measurement of Productivity Change. *Journal of Productivity Analysis*, *7*, 329 340.

Lovell, C. A. K. & Sickles, R. C. (1983). Testing Efficiency Hypotheses in Joint Production. *Review of Economics and Statistics*, *65(1),* 51-58.

Mbaga, M., Romain, R., Larue, B., & Lebel, L. (2000). *Assessing Technical Efficiency of Quebec Dairy Farms*. University of Laval, Centre for Research in the Economics of Agrifood. Rearch Series SR.00.10.

Melfi, C. A. (1984). *Estimation and Decomposition of Productive Efficiency in a Panel Data Model: An Application to Electric Utilities*. Unpublished Doctoral Dissertation University of North Carolin.

Meeusen, W., & van den Broeck, J. (1977). Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error. *International Economic Review, 18,* 435-444.

Moschini, G. (1988). The Cost Structure of Ontario Dairy Farms. *Canadian Journal of Agricultural Economics, 36,* 187-206.

Parikh, A. & Shah, K. (1995).Various Approaches to Measurement of Technical Efficiency in North-West Frontier Province of Pakistan. *Journal of Applied Economics, 12.*

Parikh, A., & Ali, F. (1995). Measurement of Economic Efficiency in Pakistani Agriculture. *American Journal of Agricultural Economics, 77.*

Pitt, M., & Lee, L. (1981). The Measurement and Sources of Technical Inefficiency in the Indonesian Weaving Industry. *Journal of Development Economics 9,* 43-64.

Richards, T. J. (1993). The Effect of Supply Management of Productivity Growth: The Case of Alberta Dairy. Unpublished Doctoral Dissertation, Stanford University, Stanford.

Richards, T. J. (1995). Supply Management and Productivity Growth in Alberta Dairy. *Canadian Journal of Agricultural Economics, 43,* 421-34.

Richards, T. J., & Jeffrey, S. R. (1996). *Cost Efficiency of Alberta Dairy Production. Staff Paper No. 96-13.* Department of Rural Economy, University of Alberta.

Richards, T. J., & Jeffrey, S. R. (1998). *Economic Performance of Alberta Dairy. Staff Paper* 98-102, Department of Rural Economy, University of Alberta.

Richards, T. J., & Jeffrey, S.R. (2000). Efficiency and Economic Performance: An Application of the MIMIC Model. *Journal of Agricultural and Resource Economics 25(1),* 232-251

Richmond, J. (1974). Estimating the Efficiency of Production. *International Economic Review, 15,* 515 - 521.

Ritter, A., & Simar, L. (1997, May). Pitfalls of Normal Gamma Stochastic Frontier Models. *Journal of Productivity Analysis, 8 (2).*

Romain, R., & Lambert, R. (1992). *Economies of Size, Technical Efficiency, and the Cost of Production in the Dairy Sectors of Quebec and Ontario, Research Series No. 21*. Groupe de recherche agro-alimentaire, Department d'Economie Rurale, Universite Laval, Quebec.

Rossi, M. (2000). Technical Change and Efficiency Measures: The Post-Privatisation in the Gas Distribution Sector in Argentina. *Energy Economics, 23*.

Rossi, M. & Canay, I. (2001). *Measuring Inefficiency in Public Utilities: Does Distribution Matter? Working Paper No.12, CEER*. Departmento de Economia y Finanzas, Universidad Argentina de la Empresa.

Schmidt, P. (1976). On the Statistical Estimation of Parametric Frontier Production Function', *Review of Economics and Statistics*, *58*, 238-239.

Schmidt, P. (1984). *An Error Structure for Systems of Translog and Share Equations*. Econometrics Workshop paper 8309. Department of Economics, Michigan State University.

Schmidt, P., & Lovell, C. A. K. (1979). Estimating Technical and Allocative Inefficiency Relative to Stochastic Production and Cost Functions. *Journal of Econometrics*, *9*, 343-366.

Schmidt, P. & Sickles, R.C. (1984). Production Frontier and Panel Data. *Journal of Business and Economic Statistics*. 2, 367-374.

Seiford, L. M. & Thrall R. M (1990). Recent Developments in DEA: The Mathematical Approach to Frontier Analysis. *Journal of Econometrics, 46*, 7-38.

Seitz, W. D. (1970). The Measurement of Efficiency Relative to a Frontier Production Function. *American Journal of Agricultural Economics*, *52*, 505-511.

Seitz, W. D. (1971). Productive Efficiency in the Steam-electric Generating Industry' *Journal of Political Economy*, *79 (4)*, 878-886.

Seyoum, E.T., Battese, G.E., & Fleming, E.M. (1998). Technical Efficiency and Productivity of Maize Producers in Eastern Ethiopia: A Study of Farmers Within and Outside the Sasakawa-Global 2000 Project. *Agricultural Economics.*

Statistics Canada (1998/1999). *Canadian Economic Observer.* Catalogue no. 11-210-XPB.

Statistics Canada (2002). *2001 Census of Agriculture.* **http://www.statcan.ca/english/Pgdb/Economy/Census/econ105j.htm**

Stevenson, R. E. (1980). Likelihood Functions for Generalised Stochastic Frontier Estimation', *Journal of Econometrics*, *13*, 57-66.

Stewart, M. (1993, December). Modelling Water Cost: further research into the impact of operating conditions on company cost, *OFWAT Research Paper number 2.*

Tauer, L. W. 1993. Short-Run and Long-Run Efficiencies of New York Dairy Farms', *Agricultural and Resource Economics Review*, *22*, 1-9.

Tauer, L. W., & Belbase, K. P. (1987). Technical Efficiency of New York Dairy Farms. *Northeastern Journal of Agricultural and Resource Economics*, *16*,10-16.

Thomas, A. C. & Tauer, L. W. (1993). Linear Input Aggregation Bias in Nonparametric Technical Efficiency Measurement. *Canadian Journal of Agricultural Economics*, *42*, 77-86.

Timmer, C. P. (1971). Using a Probabilistic Frontier Function to Measure Technical Efficiency. *Journal of Political Economy*, *79*, 776-794.

Todd, D. (1971). *The Relative Efficiency of Small and Large Firms.* Committee of Inquiry on Small Firms, Research Report No. 18 (HMSO, London).

Unterschultz, J. & Mumey, G. (1996). Reducing Investment Risk in Tractors and Combines with Improved Terminal Asset Value Forecast.*Canadian Journal of Agricultural Economics*, *44*, 295-309.

Varian, H. R. (1992). *Microeconomic Analysis* (3rd ed). New York: Norton.

Waldman, D. M. (1984). Properties of Technical Efficiency Estimators in the Stochastic Frontier Model. *Journal of Econometrics*, *25*, 353-364.

Weersink, A., Turvey, C.G., & Godah, A. (1990). Decomposition Measures of Technical Efficiency for Ontario Dairy Farms. *Canadian Journal of Agricultural Economics*, *38*, 439-456.

Western Producer, (1999). Going…Going…Gone? Can Canada's Dairy Industry Survive After Tariffs? **http://www.producer.com/articles/19…special_reports/19990708dairy3.html**

Zhu, S., Ellinger, P. N., & Shumway, C. R. (1995). The choice of Functional Form in Banking Inefficiency", *Applied Economics Letters*, *2*, 375-379.

APPENDICES – ESTIMATES CONVENTIONAL TRANSLOG AND SUMMARY STATISTICS

Appendix A – Production Frontier Estimates

Table A1 *Conventional Translog Production Frontiers*

PRODUCTION FRONTIER	VARIABLE	ESTIMATION 1 1980-1996		ESTIMATION 2 1986-1996	
		COEFF.	T-RATIO	COEFF.	T-RATIO
	CONST.	2.924*	3.033	1.457	1.125
	GC	0.190	0.685	-1.309*	-2.574
	HF	0.271	1.628	0.073*	0.333
	L	-0.154	-1.038	-0.141	-0.911
	K	-0.315	-0.956	-0.828*	-2.033
	OI	0.495	1.306	1.666*	3.079
	YR	0.036	1.331	-0.023	-0.488
	GC.YR	-0.003	-0.694	-0.004	-0.455
	HF.YR	0.001	0.528	0.001	0.161
	L.YR	-0.005*	-2.254	0.000	-0.166
	K.YR	-0.019*	-4.347	-0.024*	-2.777
	OI.YR	0.012*	2.188	0.019	1.928
	YR.YR	0.000	-0.308	0.001	1.414
	GC.HF	0.024	0.771	0.017	0.446
	GC.L	-0.008	-0.286	-0.008	-0.246
	GC.K	0.217	4.076	0.128	1.660
	GC.OI	-0.140	-2.662	0.227	2.298
	GC.GC	0.030	2.216	-0.185	-4.109
	HF.L	0.027	1.958	0.016	0.995
	HF.K	-0.052	-1.679	-0.087*	-2.573
	HF.OI	-0.037	-1.051	0.032	0.739
	HF.HF	-0.037	-3.538	-0.034*	-2.716
	L.K	0.018	0.653	0.010	0.314
	L.OI	0.018	0.646	0.013	0.404
	L.L	0.007	0.779	0.007	0.695
	K.OI	0.053	0.962	0.099	1.412
	K.K	0.004	0.105	0.052	1.273
	OI.OI	-0.057	-1.288	-0.197*	-3.279
	σ	0.094*	4.024	0.056*	3.097
	γ	0.881*	28.132	0.909*	32.525
T.E. MODEL	CONST.	-0.039	-0.267	0.184	1.493
	GH	-0.133	-3.174	-0.153*	-2.716
	KL	0.047*	2.615	0.024	1.309
	BE	-0.017*	-3.203	-0.005*	-2.598
	YR	0.008	1.208	-0.001	-0.116
	YF	Na	na	0.007*	2.611
	HS	-0.001*	-2.395	-0.001	-1.761

*Coefficient is significant at 5% level.

*The abbreviations for the variables are as follows:

Production Frontier:

GC – Grains and Concentrates; HF – Hay and Forage; L – Labour; K – Capital; OI – Other Inputs;YR – Time trend; and Interaction terms.

Technical Efficiency (TE) model:

GH – the ratio of Grains and Concentrates to Hay and Forage; KL – the ratio of Capital to Labour; BE – Breeding and Veterinary services; YR – Time trend; YF – Years of Farming; HS – Herd size.

Table A2 *Output Elasticity wrt. Inputs, Conventional Translog Production Frontiers*

INPUT	**ESTIMATION 1 (1980-1996)**	ESTIMATION 2 (1986-1996)
Grains and Concentrates	0.31	-1.13
Hay and Forage	0.20	0.02
Labour	-0.09	-0.10
Capital	-0.07	-0.63
Other Inputs	0.33	1.84
Returns to scale (RTS)	0.68	-0.004

Table A3 TE *Model From Conventional Translog Production Frontiers*

	COBB-DOUGLAS			
	Estimation 1 (1980-96)		Estimation 2 (1986-96)	
VARIABLE[+]	COEFF.	T-RATIO	COEFF.	T-RATIO
CONST.	-0.039	-0.267	0.184	1.493
GH	-0.133*	-3.174	-0.153*	-2.716
KL	0.047*	2.615	0.024	1.309
BE	-0.017*	-3.203	-0.005*	-2.598
YR	0.008	1.208	-0.001	-0.116
YF	na	na	0.007*	2.611
HS	-0.001*	-2.395	-0.001	-1.761

*Coefficient is significant at 5% level.

[+] The abbreviations for the variables are as follows GH – the ratio of Grains and Concentrates to Hay and Forage; KL – the ratio of Capital to Labour; BE – Breeding and Veterinary services; YR – Time trend; YF – Years of Farming; HS – Herd size.

Table A4 *TE measures from Conventional Translog Production Frontiers*

	CONVENTIONAL TRANSLOG	
	Estimation 1 (1980-96)	Estimation 2 (1986-96)
Mean EE	0.91	0.87
Median	0.93	0.90
Max. Value	0.99	0.98
Min. Value	0.51	0.49
Variance	0.004	0.01

Table A5 *Likelihood Ratio (LR) Tests of Hypotheses for Parameters of the Conventional Translog Stochastic Production Frontiers for Alberta Dairy Farmers*

		CONVENTIONAL TRANSLOG	
HYPOTHESIS		Estimation 1 (1980-96)	Estimation 2 (1986-96)
a) $H_0: \gamma=0$	LLF_u	566.14	431.87
Estimated frontier not different from OLS (average response) function.	LLF_R	524.70	378.33
	λ_{LL}	82.88	107.09
	Critical Value (5% Level)	14.85*	13.4*
	Decision	Reject H_0	Reject H_0
c) (i) $H_0: \delta_1=\delta_2=\ldots=\delta_5=0$ (Estimation 1)	LLF_u	566.14	431.87
(ii) $H_0: \delta_1=\delta_2=\ldots=\delta_6=0$ (Estimation 2)	LLF_R	554.05	414.25
All parameters on the variables explaining technical efficiency are simultaneously equal to zero (i.e., no TE effects)	λ_{LL}	24.18	35.24
	Critical Value (5% Level)	11.07	12.59
	Decision	Reject H_0	Reject H_0

* Critical Values are obtained from Kodde and Palm (1986). These values entail a mixed χ^2 distribution. Because $\gamma=0$ lies on the boundary of the parameter space for γ, the LR statistic for testing if H_0: $\gamma=0$ is true has asymptotic distribution that is a mixture of Chi-square distributions (Coelli et al. 1998).

Appendix B – Cost Frontier Estimates

Table B1 *Parameter Estimates of conventional Translog Cost Frontiers*

	ESTIMATION 1 1980-1996		ESTIMATION 2 1986-1996	
VARIABLE	COEFF.	T-RATIO	COEFF.	T-RATIO
Constant	5.481*	3.626	7.717*	8.487
Output (Q)	-1.082	-1.444	-2.284*	-4.896
Price of				
Grains and Concentrates (PGC)	2.004*	6.019	1.359*	2.972
Hay and Forage (PHF)	-1.045*	-3.600	-0.342	-0.784
Capital (PK)	0.264	1.220	-0.090	-0.390
Other Inputs (POI)	-0.278*	-3.143	-0.075	-0.636
Year (YR)	0.033	1.512	0.110*	2.465
Q.YR	-0.003	-0.550	-0.011	-1.058
PGC.YR	0.003	0.783	-0.002	-0.254
PHF.YR	-0.004	-1.804	-0.010	-1.988
PK.YR	-0.008	-1.231	0.015	1.075
POI.YR	0.009	1.335	0.002	0.155
YR.YR	0.000	0.079	-0.004*	-2.985
Q.PGC	-0.304*	-2.725	-0.334	-2.714
Q.PHF	0.234	3.162	0.035	0.293
Q.PK	-0.016	-0.330	0.022	0.445
Q.POI	0.065	0.596	0.326*	2.428
Q.Q	0.229*	2.544	0.514*	5.378
PGC.PHF	-0.103*	-2.232	-0.076	-1.160
PGC.PK	0.020	0.994	-0.004	-0.159
PGC.POI	0.124	1.368	-0.067	-0.643
PGC.PGC	-0.009	-0.177	0.056	0.894
PHF.PK	0.008	0.424	0.007	0.349
PHF.POI	-0.025	-0.457	-0.055	-0.723
PHF.PHF	0.055*	2.557	0.060*	2.207
PK*.POI	-0.061	-1.635	-0.090*	-2.208
PK.PK	0.038	1.158	0.077*	2.089
POI.POI	-0.038	-0.787	0.099	1.727
σ	0.095*	5.908	0.087*	6.162
γ	0.844*	28.232	0.841*	25.268

Note: Prices are relative to the price of labor
* Coefficients are significant at 5% percent.

Table B2 *EE Model Estimated with Conventional Translog Cost Frontier*

	COBB-DOUGLAS			
	Estimation 1 (1980-96)		Estimation 2 (1986-96)	
VARIABLE[+]	COEFF.	T-RATIO	COEFF.	T-RATIO
CONST.	-0.018	-0.142	-0.167	-1.012
GH	-0.054	-1.769	0.001	0.034
KL	-0.222*	-14.783	-0.252*	-11.699
BE	0.006*	4.751	0.006*	4.680
YR	-0.010	-1.610	-0.026*	-2.383
YF	n.a.	n.a.	0.010*	4.473
HS	0.001	1.753	0.001	1.645

*Coefficient is significant at 5% level.

[+]The abbreviations for the variables are as follows GH – the ratio of Grains and Concentrates to Hay and Forage; KL – the ratio of Capital to Labour; BE – Breeding and Veterinary services; YR – Time trend; YF – Years of Farming; HS – Herd size.

Table B3 EE *Measures from Conventional Translog Cost Frontiers*

	CONVENTIONAL TRANSLOG	
	Estimation 1 (1980-96)	Estimation 2 (1986-96)
Mean EE	0.83	0.84
Median	0.86	0.87
Max. Value	0.98	0.98
Min. Value	0.33	0.32
Variance	0.012	0.013

Table B4 *Likelihood Ratio (LR) Tests of Hypotheses for Parameters of the Conventional Translog Stochastic Cost Frontiers for Alberta Dairy Farmers*

		CONVENTIONAL TRANSLOG	
HYPOTHESIS		Estimation 1 (1980-96)	Estimation 2 (1986-96)
a) H_0:γ=0			
	LLF_U	285.51	214.74
Estimated frontier not different from OLS (average response) function.	LLF_R	193.48	123.29
	λ_{LL}	184.06	182.89
	Critical Value (5% Level)	14.85*	13.4*
	Decision	Reject H_0	Reject H_0
c) (i) H_0:$\delta_1=\delta_2=\ldots=\delta_5=0$ (Estimation 1)	LLF_U	285.51	214.74
(ii) H_0:$\delta_1=\delta_2=\ldots=\delta_6=0$ (Estimation 2)	LLF_R	253.66	174.70
All parameters on the variables explaining technical efficiency are simultaneously equal to zero (i.e., no EE effects)	λ_{LL}	63.70	80.08
	Critical Value (5% Level)	11.07	12.59
	Decision	Reject H_0	Reject H_0

* Critical Values are obtained from Kodde and Palm (1986). These values entail a mixed χ^2 distribution. Because γ=0 lies on the boundary of the parameter space for γ, the LR statistic for testing if H_0: γ=0 is true has asymptotic distribution that is a mixture of Chi-square distributions (Coelli et al. 1998).

Appendix C – Summary Statistics

Table C1 *Summary Statistics for Production and Cost Frontier Models*

		Milk per cow	Avg. qt. of GC/cow	Wt. Qt. of HF/cow	Av. Hrs/cow	Capital/cow	Other Inputs	Avg. price of GC	Avg. price of HF	Av.Wt. Wage rate	Price of capital	Price of OI	Total cost
1980	Mean	55.98	1.77	1.88	115.29	2515.69	175.64	137.64	148.31	6.28	0.16	1.23	1708.33
	Minimum	35.82	0.56	0.57	36.02	938.87	66.84	96.27	58.22	0.68	0.13	1.23	1089.43
	Maximum	77.46	2.96	5.93	261.64	5470.18	327.09	216.37	352.11	14.56	0.23	1.23	3079.66
1981	Mean	58.38	1.91	2.15	143.84	2553.66	210.42	153.69	138.88	5.07	0.22	1.08	1858.84
	Minimum	34.92	0.53	0.59	34.59	888.30	79.45	89.38	57.86	0.45	0.18	1.08	1028.74
	Maximum	177.59	4.38	5.20	418.58	4845.47	379.65	248.07	302.45	13.04	0.30	1.08	3044.15
1982	Mean	56.70	1.86	2.24	131.74	2577.83	234.00	138.44	144.45	6.09	0.19	1.05	1899.84
	Minimum	35.67	0.69	0.43	40.74	989.69	97.64	74.42	64.08	0.30	0.15	1.05	1100.43
	Maximum	81.61	5.09	6.27	331.22	4480.44	381.48	205.73	399.79	12.07	0.29	1.05	2861.62
1983	Mean	57.36	1.84	2.23	119.05	2557.02	216.41	139.25	143.34	8.00	0.16	1.04	1960.60
	Minimum	38.56	0.59	0.49	38.39	1129.97	61.64	51.84	65.89	0.54	0.12	1.04	1095.55
	Maximum	78.75	3.03	6.09	340.87	4594.52	431.89	233.63	333.77	19.98	0.26	1.04	3879.88
1984	Mean	59.74	1.79	2.19	137.71	2696.05	214.57	162.49	173.01	6.42	0.17	1.02	1950.06
	Minimum	42.35	0.07	0.25	42.76	1196.02	74.89	89.02	64.14	0.13	0.14	1.02	1216.01
	Maximum	83.10	3.49	7.15	359.67	4834.16	368.11	243.32	424.08	15.43	0.29	1.02	3336.57
1985	Mean	63.22	1.80	1.91	142.79	2988.50	233.80	159.61	220.87	5.81	0.16	1.02	2000.24
	Minimum	38.46	0.29	0.35	36.18	1121.83	90.26	81.26	49.89	0.21	0.13	1.02	1165.52
	Maximum	91.13	3.07	6.68	374.96	5371.12	388.52	215.87	523.40	13.30	0.28	1.02	3325.38
1986	Mean	66.56	1.99	1.58	111.45	2332.74	239.16	143.62	228.99	10.07	0.18	1.00	1902.42
	Minimum	40.00	0.49	0.52	18.80	962.52	86.25	76.68	85.43	0.14	0.14	1.00	1197.43
	Maximum	96.85	3.49	3.89	360.28	3657.23	457.06	188.91	535.28	61.48	0.26	1.00	3339.63
1987	Mean	66.21	2.23	1.87	178.13	2444.87	248.65	124.45	149.36	5.91	0.17	1.00	1933.45
	Minimum	37.64	0.67	0.43	48.43	1044.32	89.22	45.42	60.16	0.30	0.12	1.00	1088.92
	Maximum	83.84	3.99	5.25	522.73	4940.38	393.22	235.89	335.52	15.13	0.26	1.00	4744.70

1988	Mean	67.61	2.11	1.79	156.17	2438.97	243.84	157.20	170.30	5.61	0.19	0.96	1945.37
	Minimum	32.84	0.87	0.44	41.66	1109.00	94.75	82.67	63.14	0.37	0.15	0.96	1022.92
	Maximum	88.05	4.20	4.69	416.60	4685.94	419.11	235.00	324.07	16.14	0.36	0.96	3694.63
1989	Mean	69.37	2.17	1.88	179.79	2509.08	272.24	175.32	188.41	4.64	0.22	0.92	2085.75
	Minimum	35.04	1.20	0.47	52.36	1288.15	107.27	103.79	68.16	0.31	0.18	0.92	1228.89
	Maximum	87.03	3.67	8.46	428.57	4801.60	462.02	318.05	353.16	14.95	0.28	0.92	3603.80
1990	Mean	69.37	2.31	2.00	152.22	2534.69	310.00	158.54	186.03	8.44	0.23	0.91	2450.20
	Minimum	34.60	0.71	0.71	33.97	1229.67	139.22	79.29	77.25	0.22	0.19	0.91	1258.24
	Maximum	93.81	4.32	5.48	409.70	4475.66	527.65	240.45	343.71	42.03	0.36	0.91	8663.37
1991	Mean	72.29	2.19	2.29	171.90	2586.06	332.48	154.68	163.27	7.54	0.19	0.92	2545.56
	Minimum	36.68	0.58	0.49	51.00	1365.86	154.61	72.45	63.92	0.49	0.15	0.92	841.56
	Maximum	102.12	3.26	5.37	450.56	4963.94	520.20	229.73	318.37	20.56	0.36	0.92	6296.29
1992	Mean	76.08	2.42	1.97	188.10	2836.32	341.71	159.78	186.16	5.13	0.17	0.92	2159.49
	Minimum	45.05	1.03	0.44	61.54	1406.76	172.23	85.29	49.01	0.37	0.13	0.92	1237.22
	Maximum	101.80	4.41	6.19	490.81	6742.58	602.22	241.02	710.61	14.65	0.31	0.92	3411.18
1993	Mean	80.24	2.54	1.71	204.72	2860.24	349.11	169.94	242.20	5.58	0.16	0.88	2304.11
	Minimum	50.98	1.13	0.35	44.45	1260.59	174.62	83.05	84.37	0.09	0.12	0.88	1141.06
	Maximum	109.17	3.86	6.05	757.72	7397.42	835.70	255.05	461.35	18.26	0.31	0.88	5140.61
1994	Mean	82.67	2.67	1.93	189.92	3036.66	372.80	186.27	197.35	7.18	0.17	0.85	2537.93
	Minimum	59.50	1.22	0.71	22.21	1418.83	228.68	86.83	72.90	0.24	0.13	0.85	1380.37
	Maximum	110.12	5.54	9.14	631.53	6741.23	664.02	257.92	412.85	67.42	0.22	0.85	7884.13
1995	Mean	82.70	2.53	1.86	197.28	2875.93	392.76	213.54	200.42	5.37	0.19	0.82	2447.87
	Minimum	57.18	1.55	0.73	44.17	1421.34	224.33	121.81	76.50	0.09	0.15	0.82	1364.78
	Maximum	103.09	3.71	9.21	746.58	5592.83	770.94	266.58	441.26	21.23	0.24	0.82	5841.37
1996	Mean	84.94	2.66	1.58	48.89	2718.40	451.90	249.08	276.31	11.72	0.17	0.79	2454.85
	Minimum	57.95	1.36	0.51	20.01	1172.26	220.96	147.46	127.23	9.02	0.12	0.79	1667.21
	Maximum	107.66	4.54	2.76	72.87	5180.75	800.11	335.78	428.67	16.85	0.39	0.79	3440.71

Table C2 *Selected Summary Statistics for Technical/Economic Efficiency Models*

		Vet.& Med.	Herd size	yrs of farming
1980	Mean	21.11	70.35	
	Minimum	0.00	24.50	
	Maximum	96.05	182.50	
1981	Mean	19.04	69.72	
	Minimum	2.17	36.50	
	Maximum	63.09	181.00	
1982	Mean	20.52	71.80	
	Minimum	1.73	31.00	
	Maximum	46.88	173.00	
1983	Mean	21.28	68.42	
	Minimum	2.39	30.00	
	Maximum	83.48	167.00	
1984	Mean	21.01	71.42	
	Minimum	2.59	34.50	
	Maximum	61.52	171.00	
1985	Mean	22.12	72.32	
	Minimum	2.81	33.50	
	Maximum	65.39	179.00	
1986	Mean	26.47	70.15	12.82
	Minimum	2.08	30.50	2.00
	Maximum	76.14	145.00	46.00
1987	Mean	27.95	78.03	16.07
	Minimum	1.76	33.50	1.00
	Maximum	70.82	268.50	66.00
1988	Mean	27.18	79.61	15.00
	Minimum	1.82	30.50	2.00
	Maximum	64.20	273.50	48.00
1989	Mean	25.97	76.35	15.14
	Minimum	2.03	26.50	2.00
	Maximum	62.34	281.50	48.00
1990	Mean	33.78	80.96	14.98
	Minimum	5.78	27.00	1.00
	Maximum	122.66	275.50	40.00
1991	Mean	30.16	85.48	14.57
	Minimum	6.39	28.00	1.00
	Maximum	75.65	269.50	39.00
1992	Mean	32.79	88.09	14.31
	Minimum	1.62	28.00	2.00
	Maximum	90.13	268.00	36.00

1993	Mean	37.25	89.72	14.66
	Minimum	4.96	27.00	1.00
	Maximum	96.01	226.50	36.00
1994	Mean	38.89	88.82	16.36
	Minimum	6.34	32.50	2.00
	Maximum	102.17	223.50	40.00
1995	Mean	38.69	94.12	15.74
	Minimum	12.83	35.00	2.00
	Maximum	97.59	230.50	38.00
1996	Mean	44.04	91.26	16.03
	Minimum	14.18	35.00	1.00
	Maximum	133.98	226.50	43.00